Quantum Simulation Possibilities with 2-d Ion Arrays in a Penning trap

John Bollinger
NIST ion storage group
Boulder, CO
Aspen Program
Workshop on Quantum Simulation/Computation with Cold Atoms and Molecules

Overarching Themes:
I: What are the key outstanding problems from condensed matter physics which ultracold atoms and molecules can address?

II: What new many-body aspects of ultracold atoms and molecules require new techniques and new perspectives, in comparison to “traditional” solid state systems? What new insight can we obtain into issues in fundamental quantum mechanics and quantum information processing?

III: What are the main challenges for simulating quantum systems and using ultracold atoms and molecules for quantum information processing? What new simulation techniques on classical computers can be brought to bear on these challenges?

IV: What is the best way to perform a quantum computation in ultracold atoms and molecules with the appropriate fidelity? How does one then interrogate such a quantum simulation or “read out” the answer from such a quantum computer?
Aspen Program
Workshop on Quantum Simulation/Computation with Cold Atoms and Molecules

**Overarching Themes:**

I: What are the key outstanding problems from condensed matter physics which ultracold atoms and molecules can address?

II: What new many-body aspects of ultracold atoms and molecules require new techniques and new perspectives, in comparison to “traditional” solid state systems? What new insight can we obtain into issues in fundamental quantum mechanics and quantum information processing?

III: What are the main challenges for simulating quantum systems and using ultracold atoms and molecules for quantum information processing? What new simulation techniques on classical computers can be brought to bear on these challenges?

IV: What is the best way to perform a quantum computation in ultracold atoms and molecules with the appropriate fidelity? How does one then interrogate such a quantum simulation or “read out” the answer from such a quantum computer?
Trapped Ions

**Rf or Paul Trap**
RF & DC Voltages

good for tight confinement and laser cooling
small numbers of particles;
quantum computing;
optical and microwave clocks; 1-d ion arrays

**Penning Trap**
(or Penning-Malmberg trap)
DC Voltages & Static B-Field

good for laser cooling larger numbers;
microwave clocks; cold plasma studies; 2-d and 3-d ion arrays

quantum simulation experiment:
Trapped Ions

**Rf or Paul Trap**
RF & DC Voltages

- good for tight confinement and laser cooling
- small numbers of particles;
- quantum computing;
- optical and microwave clocks; 1-d ion arrays

**Penning Trap**
(or Penning-Malmberg trap)
DC Voltages & Static B-Field

- good for laser cooling larger numbers;
- microwave clocks; cold plasma studies; 2-d and 3-d ion arrays

Quantum simulation experiment:
Quantum simulation possibilities with trapped ions

Effective spin systems with trapped ions
Porras and Cirac, PRL 92, 207901 (2004)

Effective spin quantum phases in systems of trapped ions
Deng, Porras, and Cirac, PRA 72, 063407 (2005)

Quantum manipulation of trapped ions in 2-d Coulomb crystals
Porras and Cirac, PRL 96, 250501 (2006)

Quantum phases on interacting phonons in an ion trap
Deng, Porras, and Cirac, PRA 77, 033403 (2008)

Mesoscopic spin-boson models of trapped ions
Porras, Marquardt, von Delft, and Cirac, PRA 78, 010101 (2008)

simulation of a 2-ion Ising chain in a transverse field

\[ H = B_x \sum_{i=1,2} \sigma_i^x + J \sigma_1^z \sigma_2^z \]

\[ B_x = |J(t)| \]

ferromagnetic order

\[ B^x (\sigma_1^x + \sigma_2^x) \]

\[ J \sigma_1^z \sigma_2^z \]
$\Omega_{rf} \sim 2\pi \times 60 \text{ MHz}$

$\omega_{\text{radial}} \sim 2\pi \times 7 \text{ MHz}$

$r_o \sim 400 \text{ m}\mu$

Internal states of $^{25}\text{Mg}^+$

$^{25}\text{Mg}^+$: $I=5/2$

$P_{3/2}$ ($S=1/2$, $L=1$)
- $[F=4,3,2]$

$P_{1/2}$ ($S=1/2$, $L=0$)
- $[F=3,2]$

$|\uparrow\rangle$ $m_f=-2$

$|\downarrow\rangle$ $m_f=-3$

$^{25}\text{Mg}^+ (s=1/2, L=0)$
- $F=2$ $S_{1/2}$
- $F=3$

$\text{^{25}Mg}^+$ laser transitions

$|\downarrow\rangle \equiv |F = 3, m_F = -3\rangle$

$|\uparrow\rangle \equiv |F = 2, m_F = -2\rangle$

quantum state of motion (harmonic oscillator)

$|\downarrow\rangle\equiv|1\rangle$

$|\uparrow\rangle\equiv|0\rangle$

$2S_{1/2}$

\[ \begin{align*}
    \left| \downarrow \right> & \equiv |F = 3, m_F = -3\rangle \\
    \left| \uparrow \right> & \equiv |F = 2, m_F = -2\rangle
\end{align*} \]

Quantum state of motion (harmonic oscillator)

Detection \((\sigma^-)\)

\(280\) nm

\(\sim 2750\) GHz

\(2\)P\(_{3/2}\)

\(\sim 200\) GHz

\(2\)P\(_{1/2}\)

\(2^5\)Mg\(^+\) laser transitions

\(1.79\) GHz

\(\downarrow\)\(\downarrow\)\(\left| 1 \right>\)

\(\left| 0 \right>\)

\(\uparrow\)\(\downarrow\)\(\downarrow\)\(\uparrow\)\(\left| 1 \right>\)

\(\left| 0 \right>\)

\(\left| \uparrow \right>\)\(\left| \downarrow \right>\)\(\left| \uparrow \right>\)\(\left| \downarrow \right>\)\(\left| \uparrow \right>\)\(\left| \downarrow \right>\)

$2^5\text{Mg}^+$ laser transitions

$|\downarrow\rangle \equiv |F = 3, m_F = -3\rangle$
$|\uparrow\rangle \equiv |F = 2, m_F = -2\rangle$

quantum state of motion (harmonic oscillator)

Detection ($\sigma^-$)

$\sim 2750 \text{ GHz}$

$\sim 200 \text{ GHz}$

\[ \begin{align*}
\begin{array}{c}
\ket{\downarrow} \equiv |F = 3, m_F = -3\rangle \\
\ket{\uparrow} \equiv |F = 2, m_F = -2\rangle
\end{array}
\end{align*} \]

\[ \begin{align*}
\begin{array}{c}
\text{quantum state} \\
\text{of motion} \\
\text{(harmonic oscillator)}
\end{array}
\end{align*} \]

\[ \begin{align*}
\begin{array}{c}
\text{Detection (}\sigma^-\text{)}
\end{array}
\end{align*} \]

Mg\(^+\) laser transitions

280 nm

\[ \begin{align*}
\begin{array}{c}
\sim 2750 \text{ GHz}
\end{array}
\end{align*} \]

\[ \begin{align*}
\begin{array}{c}
\sim 200 \text{ GHz}
\end{array}
\end{align*} \]

quantum-Ising model: \( H = J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z + B^x \sum_i \sigma_i^x \)

eff. magnetic field through resonant rf

1.7 GHz rf
quantum-Ising model: \[ H = J \sum_{\langle i, j \rangle} \sigma_i^z \sigma_j^z + B^x \sum_i \sigma_i^x \]

eff. magnetic field through resonant rf


e.g. quantum-Ising model: \[
H = J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z + B^x \sum_i \sigma_i^x
\]

1.7 GHz rf

eff. magnetic field
(global qubit-rotation)

e.g. quantum-Ising model:

\[ H = J \sum_{\langle i, j \rangle} \sigma_i^z \sigma_j^z + B^x \sum_i \sigma_i^x \]

eff. spin-spin Interaction J  
(conditional optical dipole force)

eff. magnetic field  
(global qubit-rotation)

1.7 GHz rf


e.g. quantum-Ising model:

\[ H = J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z + B_x \sum_i \sigma_i^x \]

eff. spin-spin Interaction J (conditional optical dipole force)

eff. magnetic field (global qubit-rotation)

1.7 GHz rf

\[ F = -1.5 F \]

e.g. quantum-Ising model:

\[ H = J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z + B^x \sum_i \sigma_i^x \]

- eff. spin-spin Interaction J (conditional optical dipole force)
- eff. magnetic field (global qubit-rotation)

1.7 GHz rf

e.g. quantum-Ising model:

\[ H = J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z + B^x \sum_i \sigma_i^x \]

- eff. spin-spin Interaction J
  (conditional optical dipole force)

- eff. magnetic field
  (global qubit-rotation)

1.7 GHz rf

e.g. quantum-Ising model:

\[ H = J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z + B_x \sum_i \sigma_i^x \]

eff. spin-spin Interaction J
(conditional optical dipole force)

eff. magnetic field
(global qubit-rotation)

1.7 GHz rf

e.g. quantum-Ising model:

\[ H = J \sum_{i,j} \sigma_i^z \sigma_j^z + B^x \sum_i \sigma_i^x \]

eff. spin-spin Interaction J  
(conditional optical dipole force)

eff. magnetic field  
(global qubit-rotation)

all parameters to be chosen individually:  
(e.g. amplitude, range, anti- or ferromagnetic phase …)

\[ \text{fidelity}(\text{axes}) = 88\% \pm 3\% \]
scaling up the number of ions

- many ion strings in linear rf traps: Schaetz (MPI), Monroe (U. Md.), Leibfried (NIST), …

Large scale quantum computation in an anharmonic linear ion trap, Lin, Zhu, Islam, Kim, Chang, Korenblit, Monroe, Duan, \[\text{arXiv:0901.0579}\]

- rf trap arrays: Chiaverini (LANL), Leibfried (NIST), Chuang (MIT)…

Chiaverini and Lybarger, PRA 77, 022324 (2008)

- Penning trap

optimized electrode for a bilayer honeycomb lattice; Schmeid, Wesenberg, Leibfried, \[\text{arXiv:0902.1686v1}\]
NIST Penning trap

$B = 4.5 \, \text{T}$

$^9\text{Be}^+$

$\nu_c \sim 7.6 \, \text{MHz}$

$\nu_z \sim 800 \, \text{kHz}$

$\nu_m \sim 50 \, \text{kHz}$
NIST Penning trap

Magnetic Field

$B=4.5 \ T$

$^9\text{Be}^+$

$\nu_c \sim 7.6 \ \text{MHz}$

$\nu_z \sim 800 \ \text{kHz}$

$\nu_m \sim 50 \ \text{kHz}$
NIST Penning trap

4.5 Tesla Super Conducting Solenoid
Quartz Vacuum envelope $P < 10^{-10}$ Torr
Equilibrium plasma properties


- thermal equilibrium $\Rightarrow$ rigid rotation $\omega_r$
thermal equilibrium \implies\text{ rigid rotation } \omega_r \\
\text{constant plasma density}, \\
n_0 = 2\varepsilon_0 m\omega_r (\Omega_c - \omega_r)/e^2, \\
\Omega_c = \text{cyclotron frequency} \\
\text{plasma density }\to 0\text{ over a Debye length } \lambda_D = \left[\frac{k_B T}{4\pi n_0 e^2}\right]^{1/2}
Equilibrium plasma properties


- thermal equilibrium $\Rightarrow$ rigid rotation $\omega_r$

- $T \sim 0$ $\Rightarrow$ constant plasma density,
  \[ n_o = 2e_o m \omega_r (\Omega_c - \omega_r) / e^2, \]
  $\Omega_c$ = cyclotron frequency

- plasma density $\rightarrow 0$ over a Debye length $\lambda_D = [k_B T /(4\pi n_o e^2)]^{1/2}$

- quadratic trap potential, $e\phi_T \sim m \omega_z^2 (z^2 - r^2 / 2) \Rightarrow$ plasma shape is a spheroid
Equilibrium plasma properties


- thermal equilibrium $\Rightarrow$ rigid rotation $\omega_r$

- $T \sim 0$ $\Rightarrow$ constant plasma density,
  \[ n_o = 2\varepsilon_o m \omega_r (\Omega_c - \omega_r)/e^2, \]
  $\Omega_c =$ cyclotron frequency

  plasma density $\to 0$ over a Debye length
  \[ \lambda_D = (k_B T/(4\pi n_o e^2))^{1/2} \]

- quadratic trap potential, $e\phi_T \sim m \omega z^2 (z^2-r^2/2)$ $\Rightarrow$ plasma shape is a spheroid

  aspect ratio $\alpha = z_o/r_o$ determined by $\omega_r$
Equilibrium plasma properties


- thermal equilibrium $\Rightarrow$ rigid rotation $\omega_r$

- $T \sim 0 \Rightarrow$ constant plasma density,
  \[ n_o = 2\varepsilon_o m \omega_r (\Omega_c - \omega_r)/e^2, \]
  \[ \Omega_c = \text{cyclotron frequency} \]
  plasma density $\to 0$ over a Debye length
  \[ \lambda_D = [k_B T/(4\pi n_o e^2)]^{1/2} \]

- quadratic trap potential, $e\phi_T \sim m\omega^2 z^2 (z^2-r^2/2) \Rightarrow$ plasma shape is a spheroid

aspect ratio $\alpha = z_o/r_o$ determined by $\omega_r$

- $\omega_r$ precisely controlled by a rotating electric field (rotating wall)
Experimental techniques
Experimental techniques

axial cooling beam
Experimental techniques
**Experimental Techniques**

Bragg scattering

Axial cooling beam
Experimental techniques

strobed Bragg scattering

axial cooling beam

top-view camera

B
Experimental techniques

imaging of the ion crystals
Experimental techniques

imaging of the ion crystals
Side-view image: \( N \sim 1.8 \times 10^3 \) ions, Rotating wall spin-up from \( \omega_r/(2\pi) = 30 \text{ kHz} \) to 90 kHz
Side-view image: \( N \sim 1.8 \text{ k ions} \), Rotating wall spin-up from \( \omega_r/(2\pi) = 30 \text{ kHz to 90 kHz} \)
Real-space images with planar plasmas

with planar plasmas all the ions can reside within the depth of focus
Real-space images with planar plasmas

Density $n_o$ vs. position:
- $\omega_m$
- $\Omega_c / 2$
- $\Omega_c - \omega_m$

Rotation frequency $\omega_r$

with planar plasmas all the ions can reside within the depth of focus

65.70 kHz

1 lattice plane, hexagonal order
Real-space images with planar plasmas

with planar plasmas all the ions can reside within the depth of focus

1 lattice plane, hexagonal order

2 planes, cubic order

Density \( n_0 \)

\[ \omega_m, \frac{\Omega_c}{2}, \Omega_c - \omega_m \]

Rotation frequency \( \omega_r \)
increasing rotation frequency →

Theoretical curve from Dan Dubin, UCSD

- stick-slip motion of the crystal rotation!
- frequency offset \((\omega_r - \omega_{\text{wall}})\) due to creep of 2 -18 mHz
- regions of phase-locked separated by sudden slips in the crystal orientation
- stick-slip motion due to competition between laser and rotating wall torques
- mean time between slips \(~10\) s; what triggers the slips?

**Single ion addressing?**
Present experimental effort:

- Spin squeezing through global addressing of a single plane of ions with an optical dipole force

- First step towards an Ising model simulation with > 100 ions

**Our qubit:**

\[
\begin{align*}
2s^2S_{1/2} & \quad \text{F} = 2 \\
2p^2P_{1/2} & \quad \text{F} = 1, 2 \\
2p^2P_{3/2} & \quad \text{F} = 0, 1, 2, 3
\end{align*}
\]

- Cooling
- Raman

\[
\begin{align*}
\langle +3/2, +1/2 \rangle & \quad \text{~80 GHz} \\
\langle +3/2, -1/2 \rangle & \quad \text{~40 GHz} \\
\langle +1/2, -1/2 \rangle & \quad \text{124 GHz} \\
\langle -1/2, -1/2 \rangle & \quad \text{~80 GHz}
\end{align*}
\]
Rabi flopping on 124 GHz electron spin flip

124 GHz Microwave Feed Horn

Teflon lens

6.35 cm

60 cm

-\textit{V}_0

axial cooling beam

\textbf{B}

Rabi time

\textbf{(a)}

\textbf{(b)}

Probability |\textup{\uparrow}\rangle

0 1 2

0 20 40 60 80

Rabi Time (ms)
Ramsey\( (T_2)\) coherence on 124 GHz electron spin flip

coherence can be extended to 10’s of ms with spin echo (dynamical decoupling)

Biercuk, Uys, VanDevender, Shiga, Itano, Bollinger, Nature 458, 996 (2009)
Spin squeezing through single axis twisting
Kitigawa and Ueda, PRA 47, 5138 (1993)

1. prepare $|\uparrow\uparrow\uparrow \cdots \uparrow\rangle = |J = \frac{N}{2}, M_J = \frac{N}{2}\rangle$, $T_{\text{motional}} \sim 0.5 \text{ mK}$

2. $\pi/2$ pulse of 124 GHz microwaves

$|J = \frac{N}{2}, M_J = \frac{N}{2}\rangle \rightarrow \sum_{M_J=-\frac{N}{2}}^{\frac{N}{2}} c(N, M_J) |J, M_J\rangle$
3. Apply $\exp(i\chi \{J_z\}^2 t)$ “push” gate on the axial center-of-mass mode of a single ion plane

$$\nu_L + \omega_z + \delta$$

Raman beams


$$\chi = \left(\frac{\eta \Omega}{\delta}\right)^2$$

$$t = m\frac{2\pi}{\delta}, \quad m = 1, 2, 3, ...$$

4. Measure $\Delta J_z$ as a function of rotation about x-axis (mean spin vector direction)

Microwave with 90°Phase shift
Experimental parameters for squeezing interaction $\exp(i\chi\{J_z\}^2 t)$

$\theta \sim \pm 1^\circ \Rightarrow \lambda_{\text{eff}} \sim 10 \ \mu m$

$N=100$, $\nu_z = 800 \ \text{kHz}$, $T \sim 0.5 \ \text{mK}$

$$\eta = \sqrt{\frac{\hbar}{2m\omega_z}} 2k \sin \theta, \eta_{\text{COM}} = \frac{\eta}{\sqrt{N}}$$

Lamb Dicke limit $\eta \sqrt{\bar{n}+1} \approx 0.07$

$\bar{n} \approx 12$

5 mW beams, $\text{waist}_z \approx 50 \mu m$, $\text{waist}_x \approx 580 \mu m$

(90% uniformity across $2R_p = 260 \mu m$)

$\Omega \approx 2\pi \times 140 \ \text{kHz}$ for 20 GHz Raman detuning

For optimum squeezing $\chi t \approx \eta_{\text{COM}}^2 \frac{\Omega^2}{\delta} \times \frac{2\pi}{\delta} \approx 0.04 \times \frac{\pi}{2}$

$\Rightarrow \delta \approx 2\pi \times 2.8 \ \text{kHz}$, $t = \frac{2\pi}{\delta} \approx 360 \ \mu s$, $\chi \approx 2\pi \times 30 \ \text{Hz}$

For $\theta \sim \pm 1^\circ$, actual squeezing will be limited by spontaneous emission
Evidence for optical dipole excitation of the axial COM mode with N~200 ions

Motion towards beam, fluorescence is brighter

Motion away from beam, fluorescence is dimmer
$J_z^2$ interaction permits the realization of a long range Ising interaction?

\[
J_z^2 = \left( \sum_i \sigma_i^z \right) \times \left( \sum_j \sigma_j^z \right) = 2 \times \sum_{i<j} \sigma_i^z \sigma_j^z + \sum_i (\sigma_i^z)^2
\]

This interaction can be solved analytically. Does it exhibit a quantum phase transition?

\[
H = -B_x \sum_i \sigma_i^x + \chi J_z^2
\]

Simulating this Hamiltonian could serve as a useful test of the fidelity with which we can simulate Ising models in a Penning trap

Steps for increasing $\chi$
- larger Raman beam power (x10 reasonable)
- larger Raman beam angles can help – Lamb Dicke requirement eventually means colder temperatures
- smaller Raman beam detuning (spontaneous emission?)

\[
\chi \approx \frac{\eta_{COM}^2 \Omega^2}{\delta} \sim 2\pi \times (30 \text{ Hz})
\]
Ising interaction with intermediate range (a computationally complex system) can be generated by optical dipole forces which couple to many modes of the planar array.

**Effective spin systems with trapped ions**  
Porras and Cirac, PRL 92, 207901 (2004)

For planar arrays and an adiabatic push

\[
H = -B_x \sum_i \sigma_i^x + \sum_{i<j} J_{i,j} \sigma_i^z \sigma_j^z
\]

\[
J_{i,j} \approx \frac{F^2 e^2}{(m \omega_z^2) d_{i,j}^3}, \quad F = \text{optical dipole force}
\]

\[
d_{i,j} = \text{equilibrium distance between } i, j
\]

\[
\approx \frac{F^2}{m \omega_z^2} \times \frac{e^2}{m \omega_z^2 d_{i,j}^3} \leq \frac{F^2}{m \omega_z^2} \times 0.07 \text{ for single plane}
\]

With our current parameters \(J_{i,j}\) tiny (~1 Hz)

require larger Raman beam powers, Raman beam angles

What about Raman beam detuning \(\Delta\)?  
\[ J_{i,j} \sim \frac{1}{\Delta^2}, \quad \Gamma(\text{spontaneous emission}) \sim \frac{1}{\Delta^2} \]

How small can \(\Delta\) be?
Quantum simulation with ~100 ion planar arrays looks feasible – laser power requirements are challenging

**Summary**

Quantum simulation with ~100 ion planar arrays looks feasible – laser power requirements are challenging

**Features of trapped ion simulation**

- trapped ions naturally permit the realization of quantum spin Hamiltonians
- hexagonal 2-d lattice enables the study of spin frustration
- the state of individual ions can be measured with high signal-to-noise ratio which permits direct measurement of the spin correlation functions
- trapped ion simulations can be done with small ion numbers through several hundred ions; the basic building blocks of strongly interacting systems quantum systems can be studied and understood, helping with the measurement and interpretation of larger scale systems
- general 2-d lattice symmetries can be achieved with properly designed surface electrode rf trap arrays
Slow heating at short times: 50-100 mK/s due to residual gas collisions

Ion heating rate

Jensen, et. al., PRL 94, Jan. 05; PRA 70, (2004)

How much time is there for entangling the ions?

\[ \frac{1 \text{ mK}}{50 \text{ mK/s}} \sim 20 \text{ ms} \]

longer ??

\[ N_{\text{ion}} = 440,000 \]
\[ V_{\text{trap}} = 500 \text{ V} \]
\[ f_{\text{rot}} = 64 \text{ kHz} \]

Slow heating at short times: 50-100 mK/s due to residual gas collisions
The data agrees with the theoretical dispersion relationship for drumhead waves in a plasma slab of thickness $2Z_p$.

$$kZ_p = (2\pi) \frac{Z_p}{\lambda}$$

$$\tan \left[ \frac{kZ_p}{\sqrt{\omega_p^2 / \omega^2 - 1}} \right] = \sqrt{\frac{\omega_p^2}{\omega^2} - 1}$$
drum head mode frequencies (laboratory frame, fluid model)
Weimer, Bollinger, Moore, Wineland, PRA 49, 3842 (1994)

N=100, \( \nu_z = 870 \text{ kHz} \), near 1-2 plane transition

\[ |\omega_{1,0}| = \omega_z \]

\[ |\omega_{2,1}| = \omega_z +/- \omega - (1/8)\pi \alpha \omega_z \]

\[ |\omega_{3,0}| = \omega_z [1-(5/16)\pi \alpha \omega_z] \]

\[ \frac{1}{8} \pi \alpha \omega_z = 2.579 \times 10^4 \]

\[ \frac{5}{16} \pi \alpha \omega_z = 6.446 \times 10^4 \]

\[ \frac{\omega_r}{2\pi} - \frac{1}{8} \pi \alpha \omega_z = 3.04 \times 10^4 \]

\[ \frac{\omega_r}{2\pi} + \frac{1}{8} \pi \alpha \omega_z = 8.197 \times 10^4 \]

\[ \alpha = \frac{Z_p}{R_p} = 0.075 \]