# String Theory:
# Coming to a Laboratory near You?

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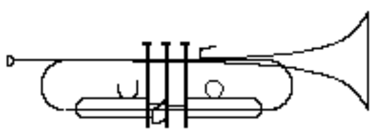
Not for wide distribution!
Warning: Prepared at last minute to fill in for ill speaker!!

*Aspen, 1st June 2009*



# Some Reviews and guide to literature:

**General AdS/CFT technology:**

O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri,
and Y. Oz, Phys. Rept. 323, 183 (2000), hep-th/9905111

**Applications to Heavy Ion collisions; Quark-Gluon plasma:**

M. Natsuume (2007), hep-ph/0701201

**Applications to Condensed Matter Physics:**

S. A. Hartnoll (2009), 0903.3246

C. P. Herzog (2009), 0904.1975

# The Goal

We need to understand various new types of behaviour

Various kinds of phase transition...

These phases are interesting in their own right...

...but may have a lot to tell us about Physics in ultra-extreme conditions in Nature

Hot, dense soup of Quarks and Gluons

Cold droplet of Lithium atoms

Exciting and Novel!

How often do we create new phases of matter in the lab?!

early universe...

cores of compact stars...

new materials....

# Understanding the Physics

How to understand all this?

Need robust models of these properties for these new forms of matter.

Is a description in terms of molecules best for getting to grips with wetness of water?

Big Question...

Is a model of quarks and gluons the best starting point?

Is a model of individual atoms/ fermions the best starting point?

Are these the most natural variables to describe the physics?

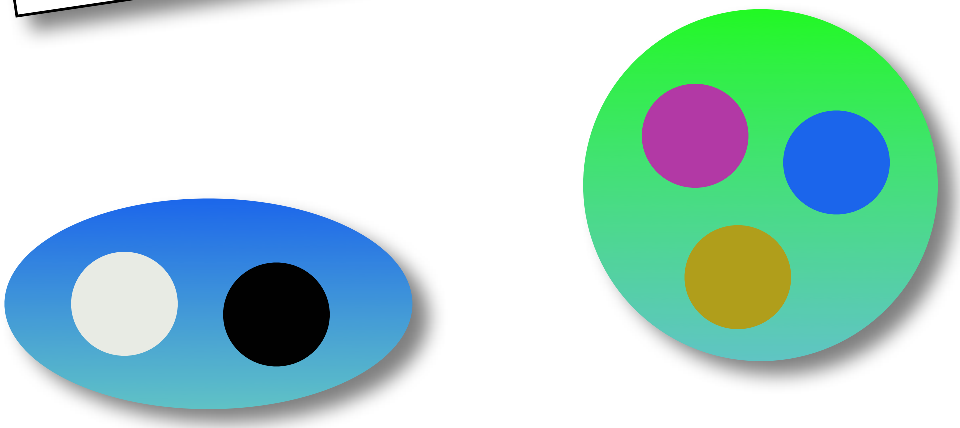
Perhaps the answer is no...

# Part I

## Cooking with Quarks and Gluons:
## Recipes from the String Theory Kitchen

# The Familiar

Ordinary nuclear matter...

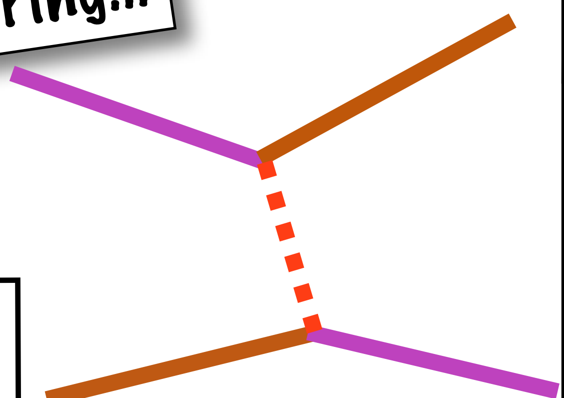


Quarks bound together by confining strong nuclear force (gluon exchange)

Low to medium energy hadronic physics

Phenomenon of confinement manifest...

High energy scattering...



quarks interact by gluon exchange

Reasonably well understood in terms of quantum field theory...

## Quantum ChromoDynamics (QCD)

asymptotic freedom, etc...

# The Unfamiliar

What happens at High Temperatures and Densities?

Lots of ideas...

Sharp change in the number of degrees of freedom

Expectations from QCD

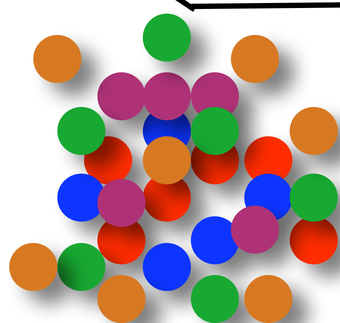
High energy per quark and gluon...

**High Temperature**

Asymptotic freedom suggests interaction weak...

Form a non-interacting gas...?

**Quark-Gluon Plasma**



Deconfining Phase Transition!

# The Unfamiliar

What happens at High Temperatures and Densities?

Lots of ideas...

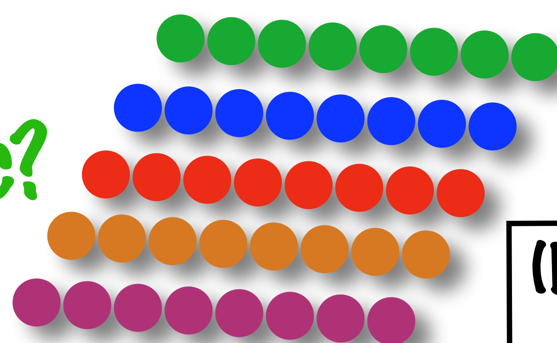
Expectations from QCD

**High Density**

Phase space very different...high momentum, so AF again suggests weak interaction

Quantum effects probably very important.

**Crystals? Colour Superconductivity, etc?**

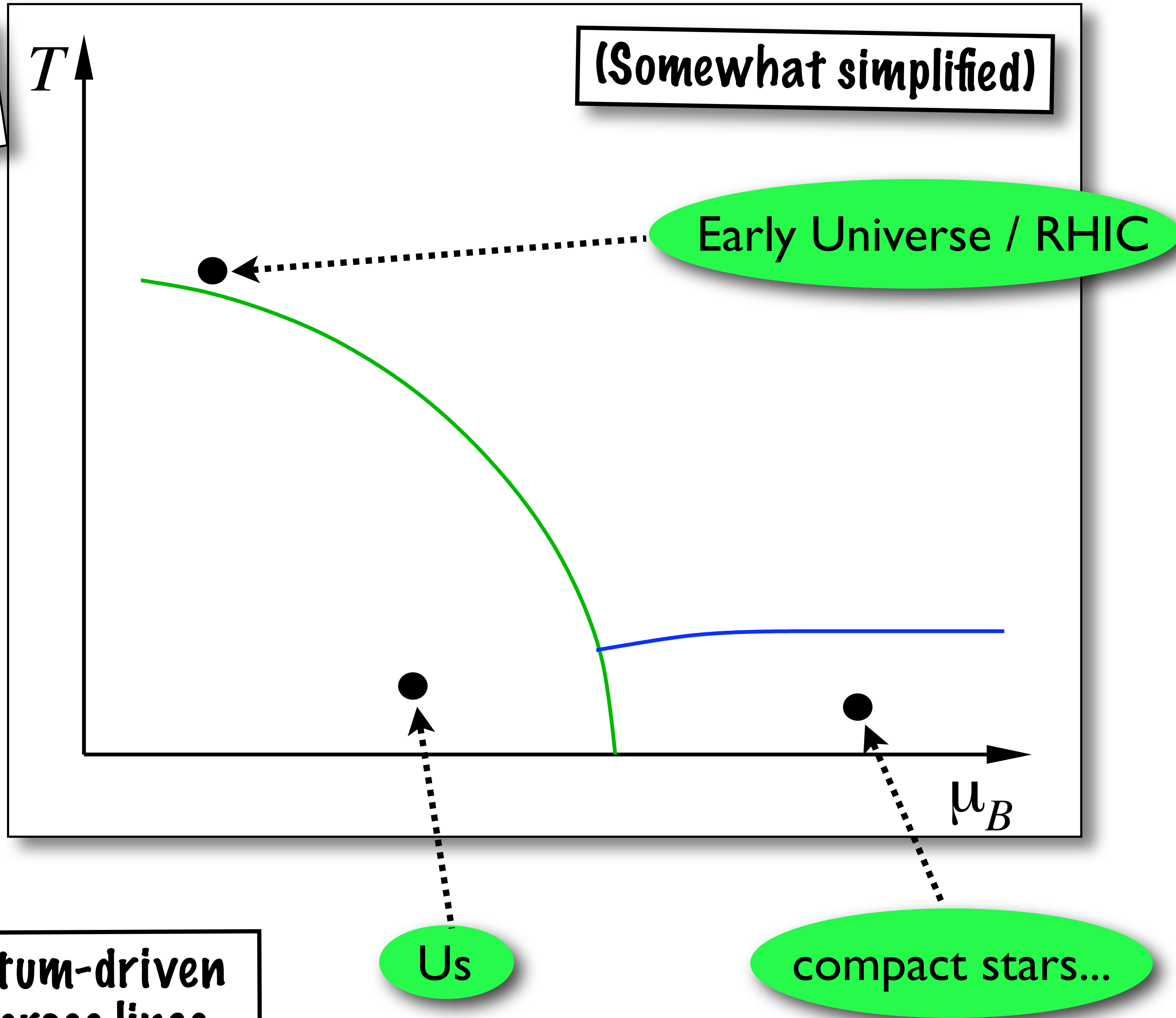


Sharp reorganization of the degrees of freedom

(Metaphor for Condensed matter type quantum-driven phase structure)

# QCD Phase Diagram?

Sketch of expectations...




Thermally or quantum-driven phase transitions across lines...

# Physics of the Plasma

If really free...

$$\varepsilon = \sigma_{SB} T^4$$

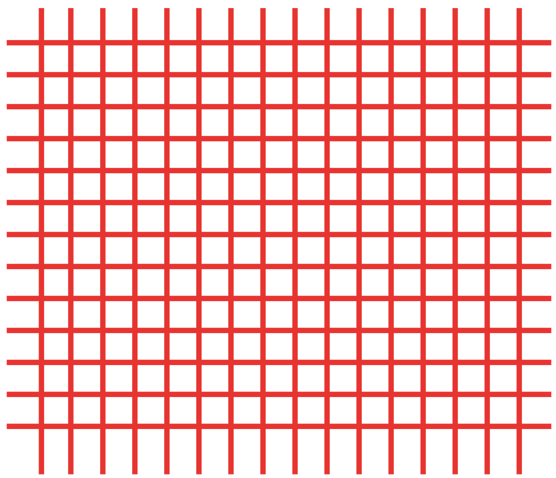
The basic result from thermodynamics...

Expect some corrections to this. Need to do a real computation in QCD...

Non-perturbative QCD can be done in this regime by numerical computations...

"Lattice Gauge Theory"

Spacetime is discretized, so is the field theory...

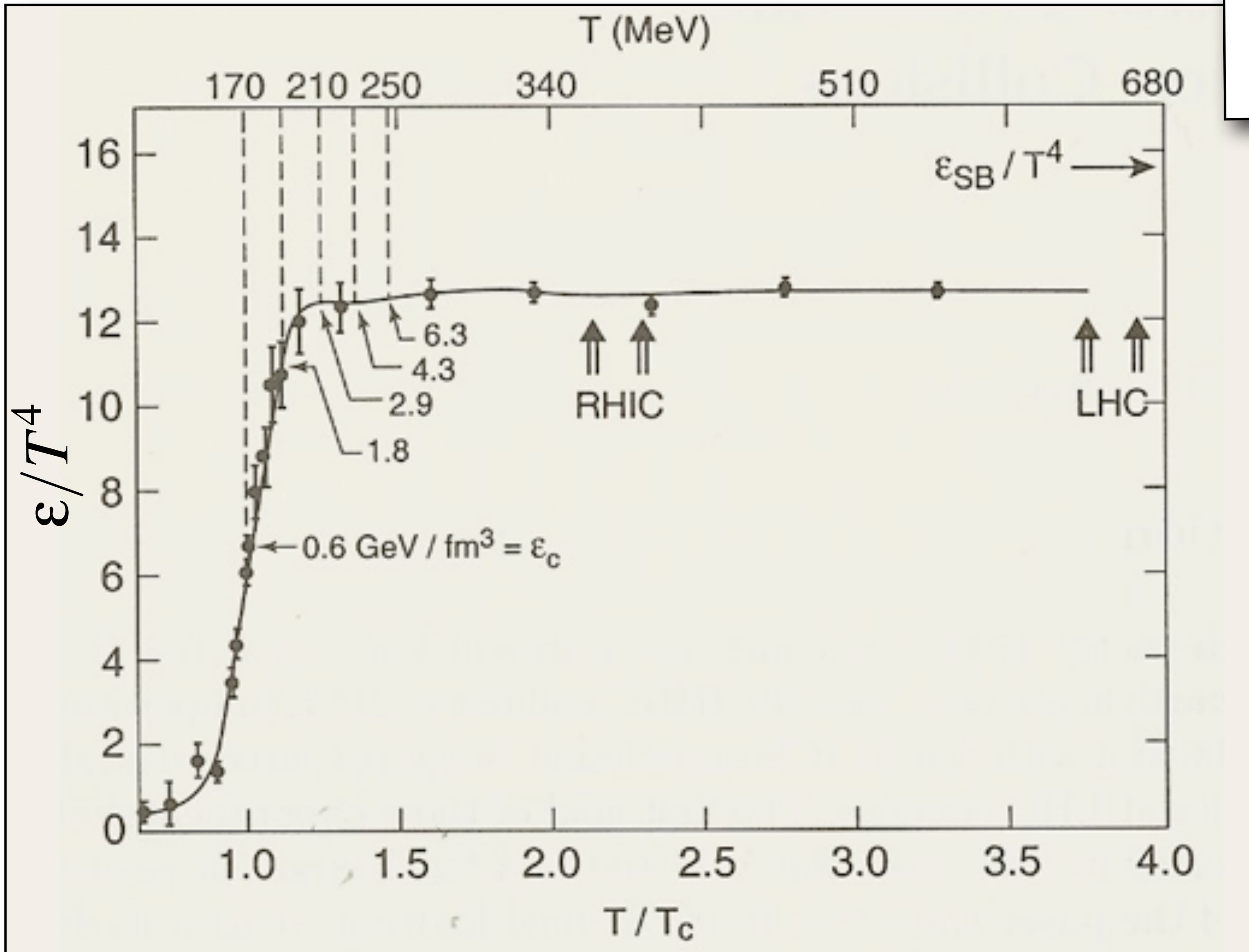


Results then extracted in the continuum limit...

# Physics of the Plasma

From:C. Lourenco in "Lectures on Quark Matter", Springer Lecture notes on Physics, 583/2002

**Results:**




Notice: Very striking in all examples...

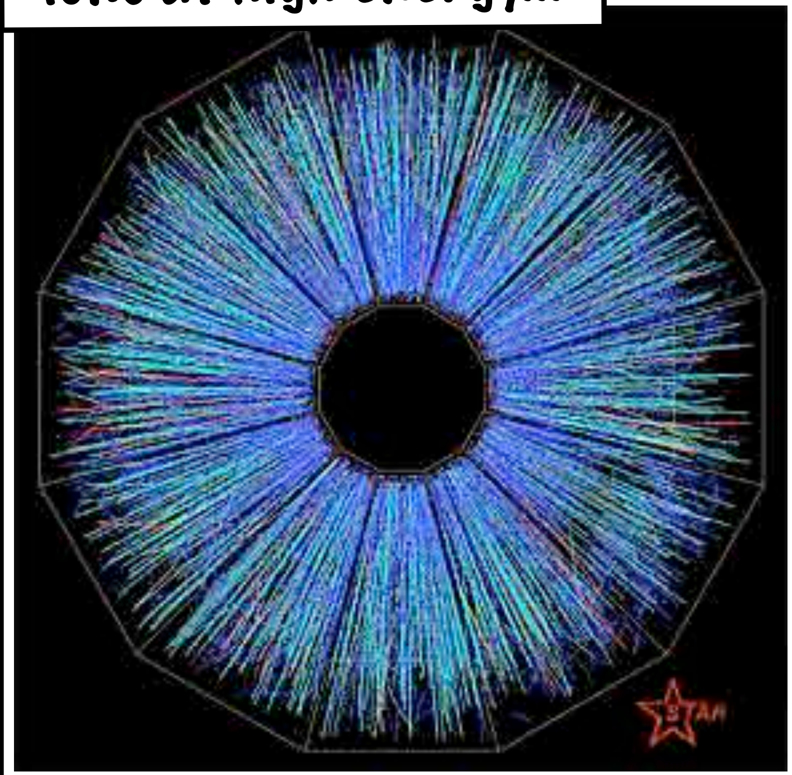
$$\varepsilon \sim 0.8\, \varepsilon_{SB}$$

Plasma is deconfined, but not actually free.

# Physics of the Plasma

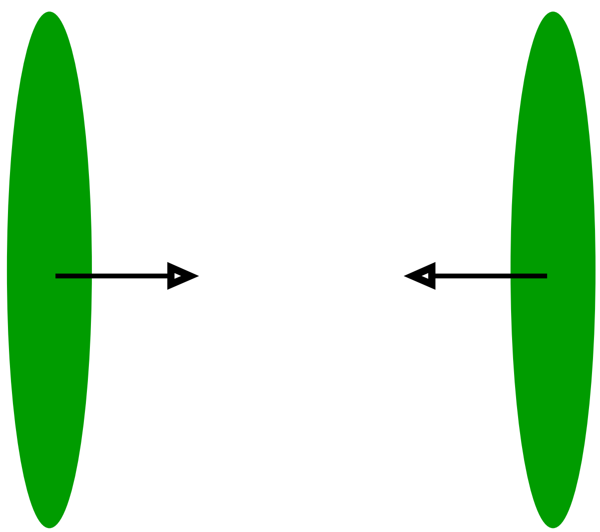
RHIC started doing experiments to probe plasma's properties directly...

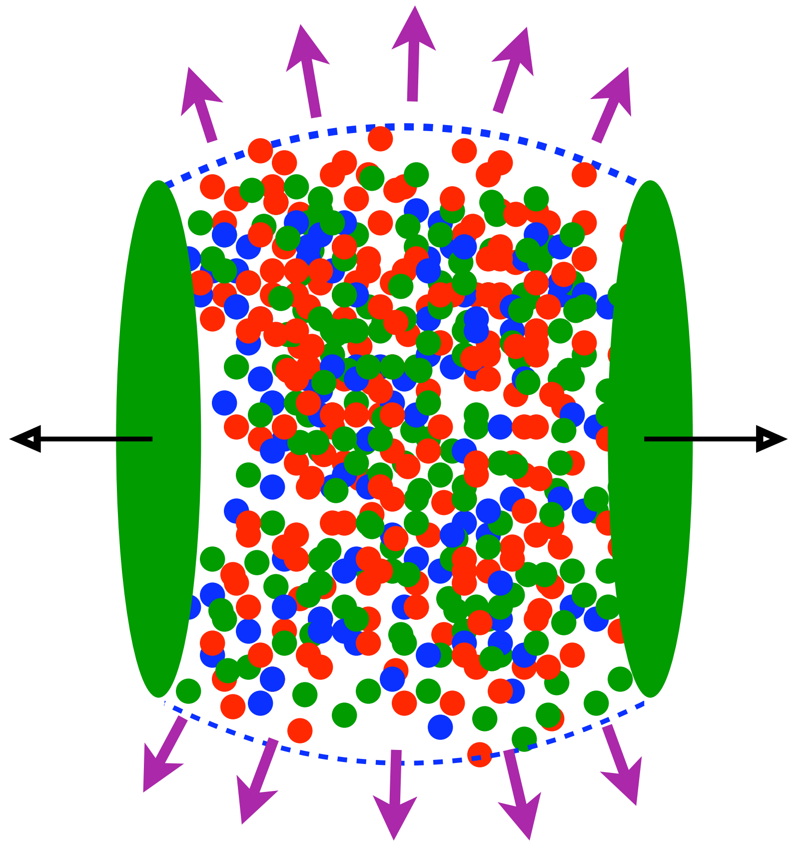

Colliding together gold ions at high energy...



# Physics of the Plasma

RHIC hydrodynamic modeling suggests that the plasma is more like a strongly interacting fluid...





Other properties include:

Shear viscosity to entropy ratio extraordinarily low $\frac{\eta}{s} < 1$

High degree of jet-quenching (lots of energy of a heavy quark absorbed by plasma)

# Understanding the Plasma

How to understand all this?

Need robust models of these properties for this new form of matter.

Big Question...

**Is starting with a model of quarks and gluons the best starting point?**

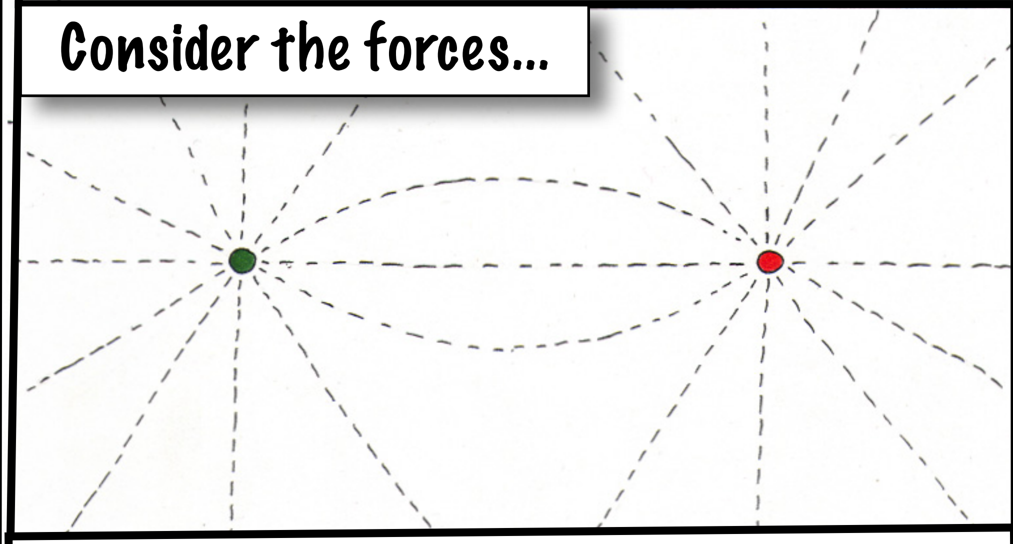
Are these the most natural degrees of freedom for this regime?

?

Perhaps the answer is no...

# Another Possibility

Consider the forces...

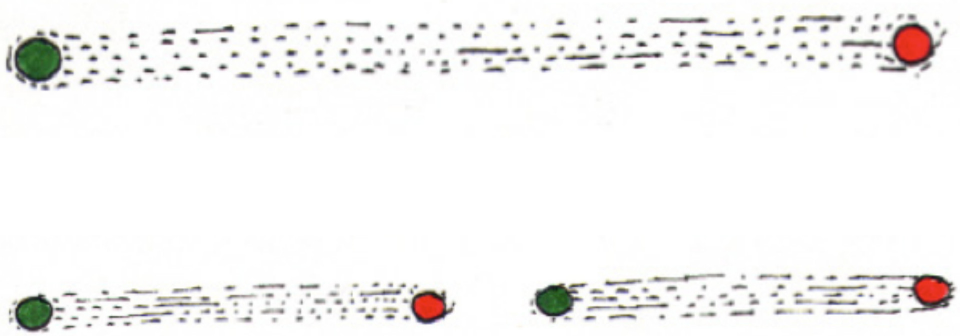


Can pull apart objects that interact electromagnetically until they are essentially free...

Is this all a clue?

Perhaps the right variables come from strings!!

But not for quarks interacting with the strong nuclear force...

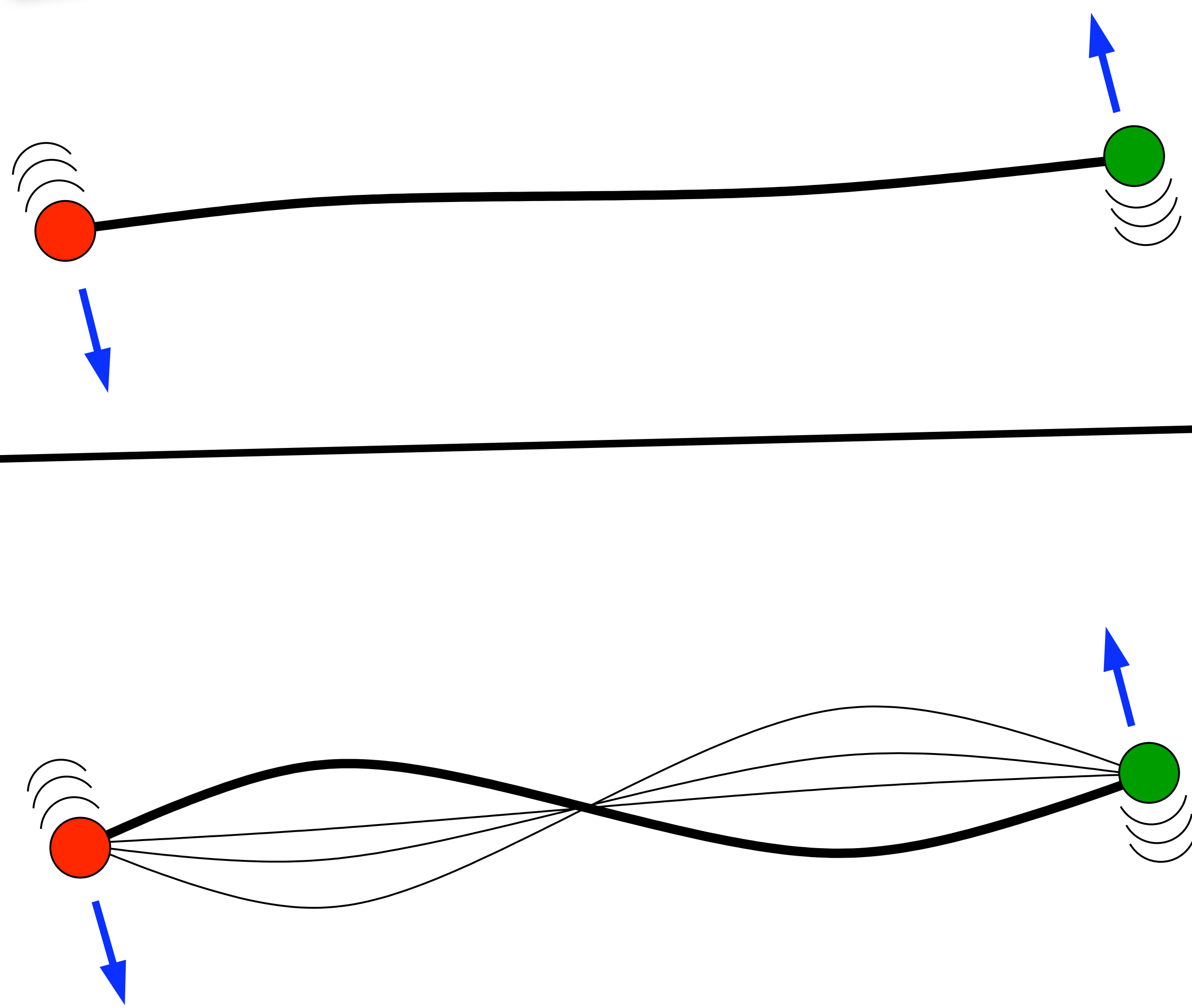


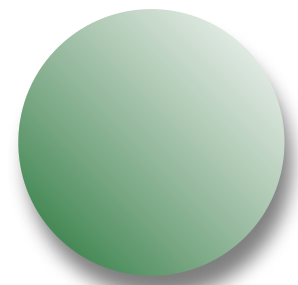
Goes back to the old ideas...

# History 1:The Old String Idea

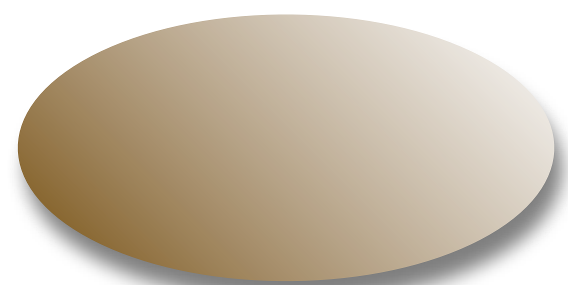
The old ideas from the 60s and early 70s...

...different nuclear particles correspond to different vibrations and spinnings of the string...





meson...



some other meson...

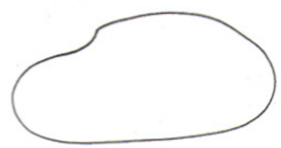
# History 2: Strings Fail!

The physics of strings is very intricate. In order to work, the theory presents a list of demands:

There must be open strings and their vibrations...
There must be closed strings and their vibrations...
There must be more than three spatial dimensions...

There must be open strings and their vibrations...

There must be closed strings and their vibrations...



There must be more than three spatial dimensions...

Ok. That's where we came in. Works pretty well.

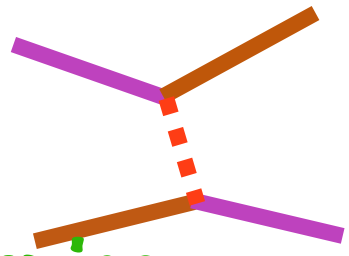
They describe a strange non-nuclear particle

Ok.. that's the last straw...!

So clearly this is just wrong... right?

# History 3: Quarks and Gluons Rule

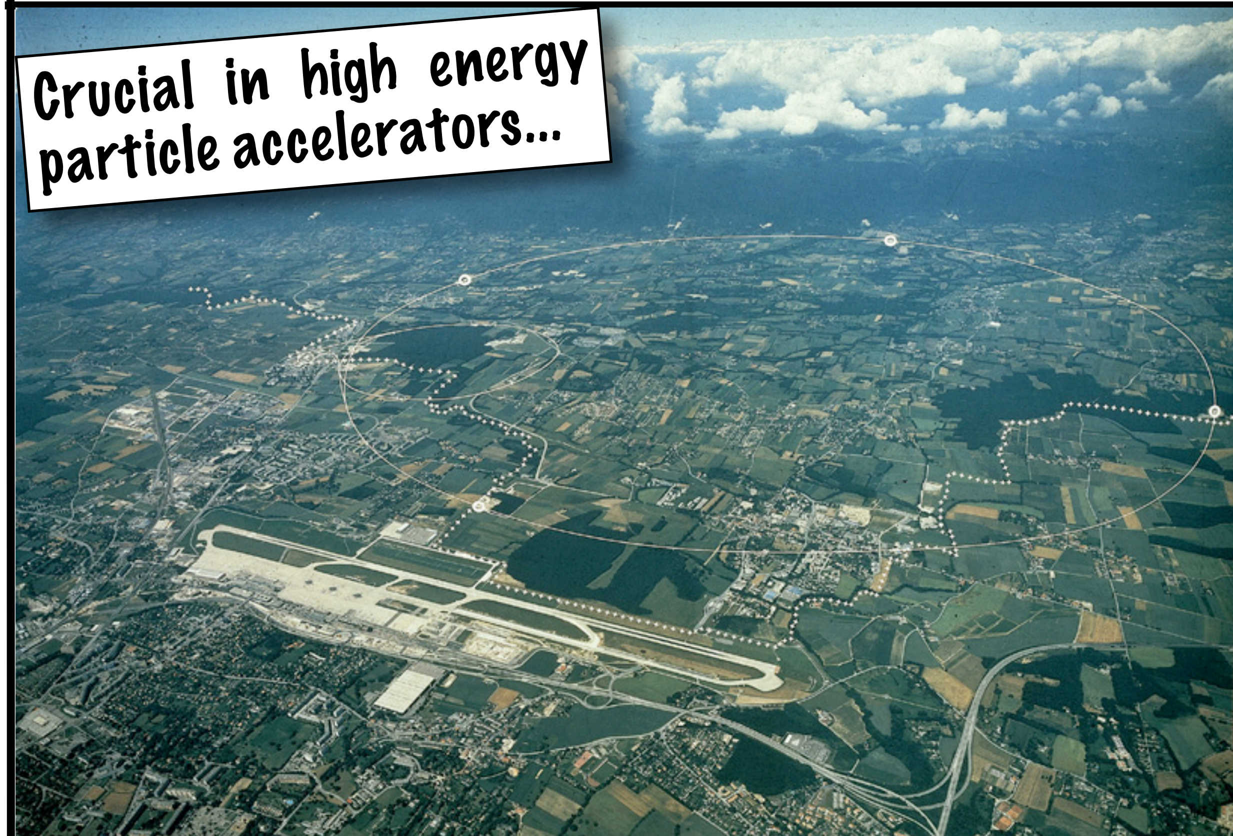
These bugs - together with the QCD approach - killed the string attempt, in the 70s.



Quantum ChromoDynamics (QCD)

Powerful for studying many aspects of quark-gluon physics

Crucial in high energy particle accelerators...



But these techniques fall short of what we need now...

# Recipe #1: Fun with Glue

**Ingredients:**

Five Spacetime Dimensions
Gravity
A negative cosmological constant

$$S = \frac{1}{16\pi G_5}\int d^5x\,\sqrt{-g}(R-2\Lambda)$$

$$\Lambda = -\frac{6}{\ell^2}$$

**Method:**

Place it all in a container that is aymptotically Minkowski on the boundary.

$$ds^2 = \frac{u^2}{\ell^2}\left(-dt^2+dx^2+dy^2+dz^2\right)+\ell^2\frac{du^2}{u^2}\,.$$

Raise the temperature from zero to $T$. The equilibrium situation will be a Schwarzschild (AdS) black hole of radius $u_0$, proportional to $T$.

(Metric displayed later.) Temp and Mass-energy of the black hole is:

$$T = \frac{u_0}{\pi\ell^2} \qquad E = \frac{3\pi u_0^4}{8G_5\ell^2}$$

Success of a meal can be all in the plating... How to serve this?

# Recipe #1: Fun with Glue

To serve:

The Newton Gravitational Constant is rewritten as:

$$G_5 = \frac{\pi \ell^3}{2N^2}$$

Divide by volume of container to get energy density:

$$\varepsilon = \frac{3}{8}\pi^2 N^2 T^4 = \frac{3}{4}\sigma_{SB} T^4$$

Whose $\sigma_{SB}$ ?

This is rather reminiscent of what we saw for the quark-gluon plasma...!

# What is Going on?

While you digest...

There is a dictionary between the five dimensional gravity quantities and a four dimensional theory!

The gravity theory is part of a larger theory... see later.

And the four dimensional theory is...?

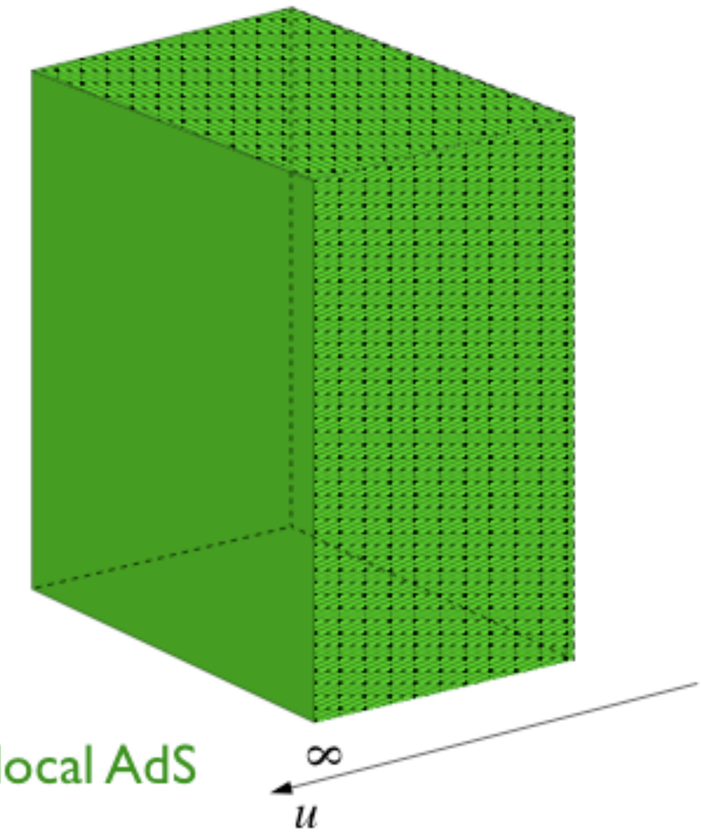
*SU(N)* Yang-Mills with 6 massless bosons and 4 massless fermions in the adjoint.

$\lambda = g_{YM}^2 N$ must be large

Glue plus some extra junk that comes along for the ride...

# More on the Dictionary

The overall picture:




local AdS

$$ds^2 = \frac{u^2}{\ell^2}\left(-dt^2 + dx^2 + dy^2 + dz^2\right) + \ell^2 \frac{du^2}{u^2}$$

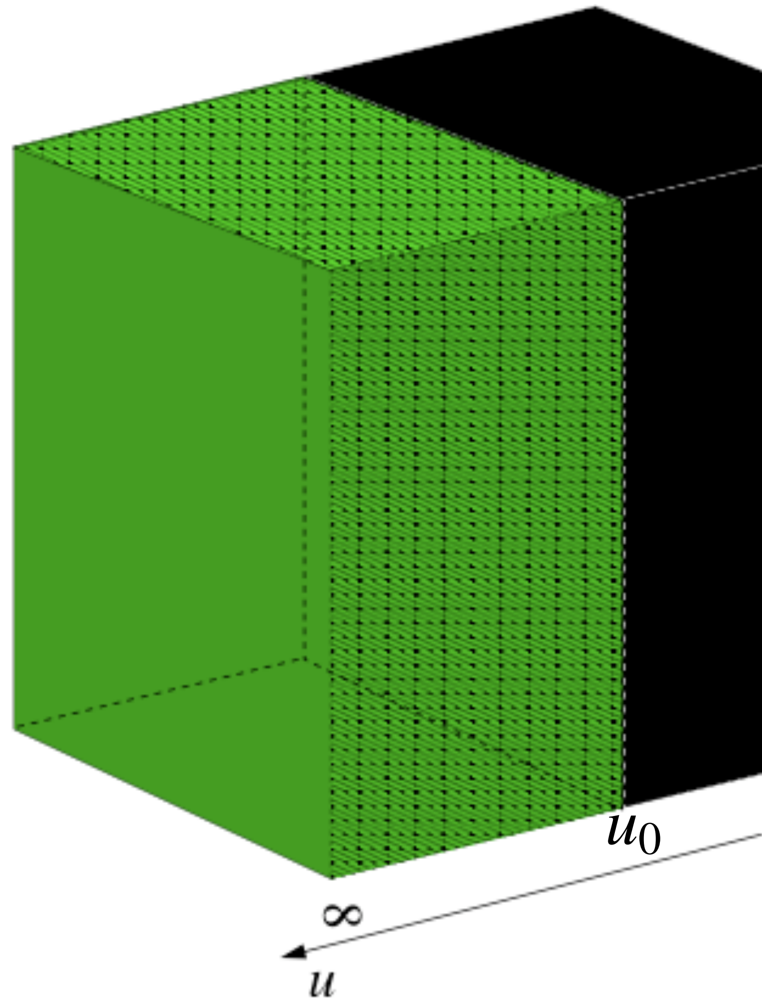
Sometimes this is a useful picture too...

Shot of "global" AdS

Field theory quantities read off using asymptotic behaviour of "bulk" fields.

# More on the Dictionary

**When at finite temperature:**




$$f(u) = \frac{u^4 - u_0^4}{u^2}$$

$$ds^2 = -\frac{f(u)}{\ell^2}dt^2 + \frac{u^2}{\ell^2}\left(dx^2 + dy^2 + dz^2\right) + \ell^2\frac{du^2}{f(u)} \,.$$

**Black hole horizon at finite radius.**

# Dessert for Recipe #1

For example, the stress tensor of the field theory couples to the graviton in bulk...

Shear viscosity in the field theory is part of a two-point function for the stress tensor...

This translates into an absorption cross-section for scattering a graviton off the black hole, which is given by the area!

$$\eta = \frac{A}{16\pi G}$$

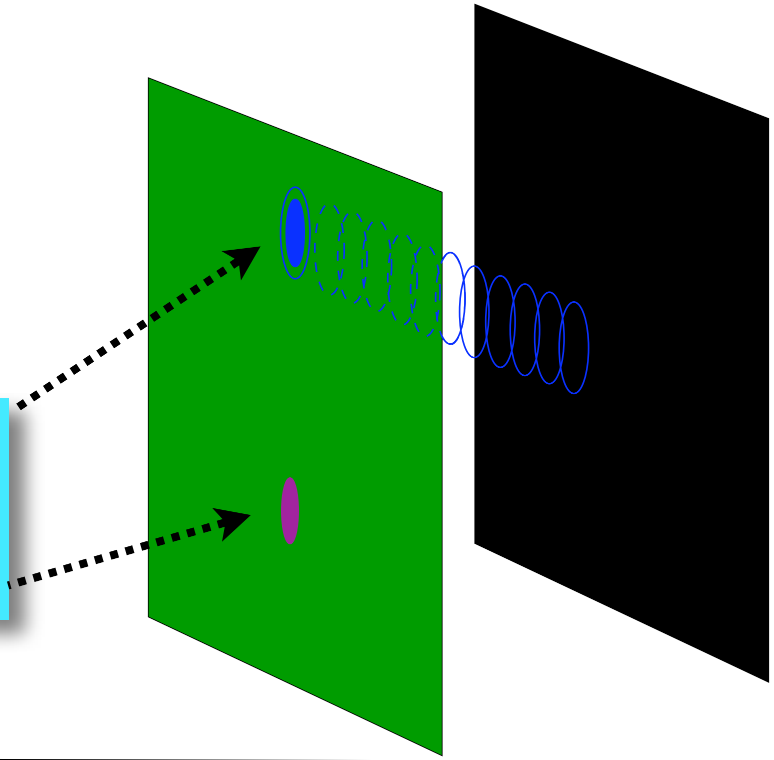
$$S = \frac{A}{4G}$$

$$\frac{\eta}{S} = \frac{1}{4\pi}$$

cartoon of this follows, over...

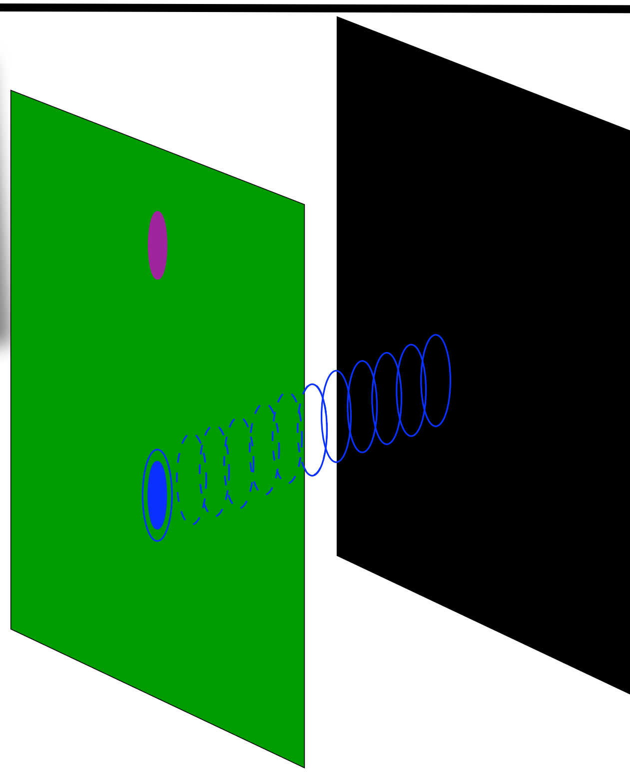
Computation of the Viscosity...

See how two separated points in the plasma connect to each other...



The disturbance travels as closed strings (gravitons) in the higher dimensions...

...and scatters off the black hole before returning!



Bigger black hole absorbs more, reduces disturbance, increasing viscosity.

viscosity

$$\eta = \frac{A}{16\pi G}$$

entropy

$$S = \frac{A}{4G}$$

$$\frac{\eta}{S} = \frac{1}{4\pi}$$

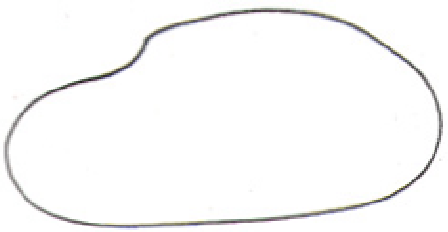
Close to what's been seen!

# What's Going On?

This is a fully quantum theory gravity we are working with here - it is embedded in string theory.

Ah, so glue really is modelled by closed strings...

Gravity is carried by the closed strings in the theory.



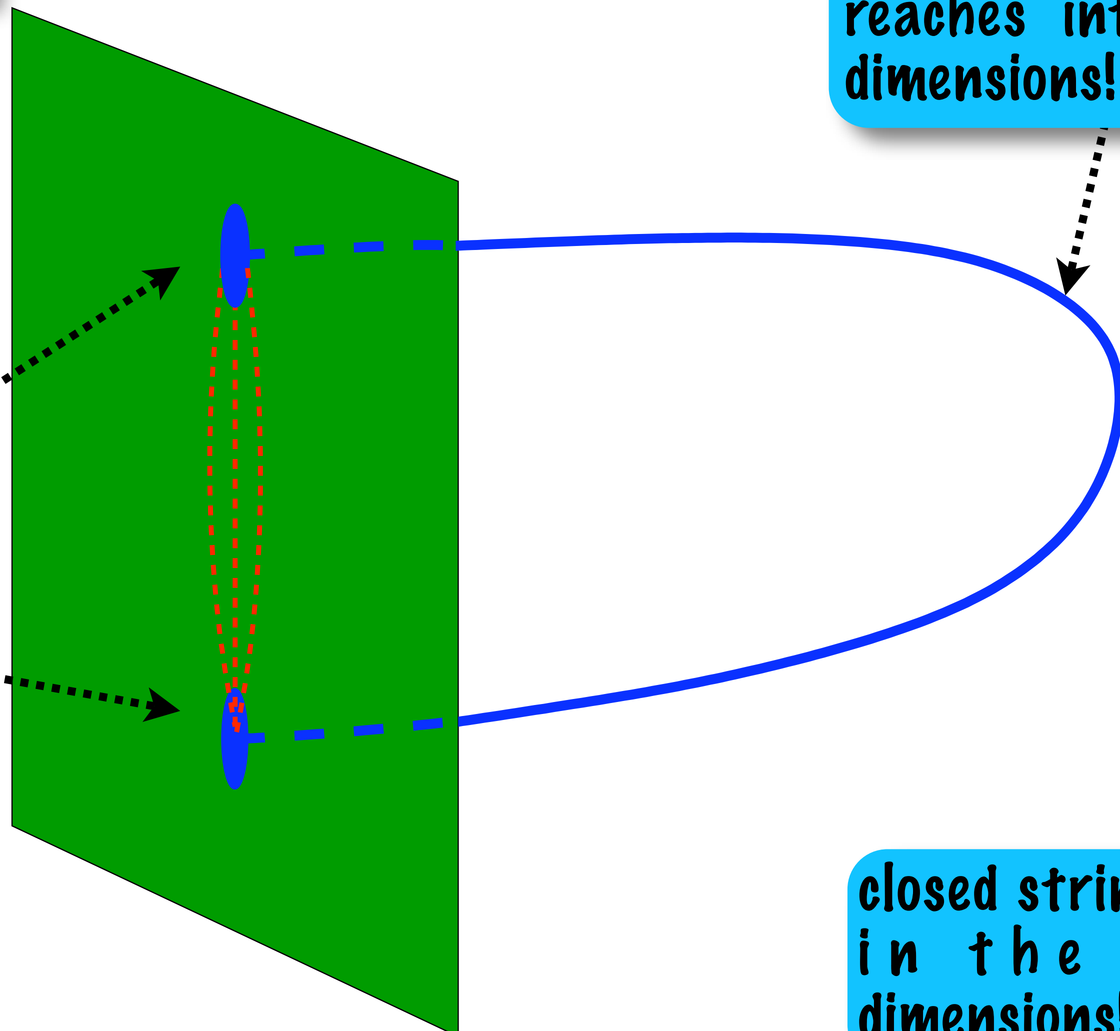
The loophole was that the gravity is in higher dimensions...!

In fact, the string theory is ten dimensional (Type IIB), and the full theory is on:

$$\mathrm{AdS}_5 \times S^5$$

$$ds^2 = \frac{u^2}{\ell^2}\left(-dt^2 + dx^2 + dy^2 + dz^2\right) + \ell^2\frac{du^2}{u^2} + \ell^2\left(d\theta^2 + \cos^2\theta d\Omega_3^2 + \sin^2\theta d\phi^2\right) \ .$$

The old ideas get to work after all. The bugs became features...!



quarks are ends of strings, living in the three space dimensions...

But the string that connects them reaches into extra dimensions!

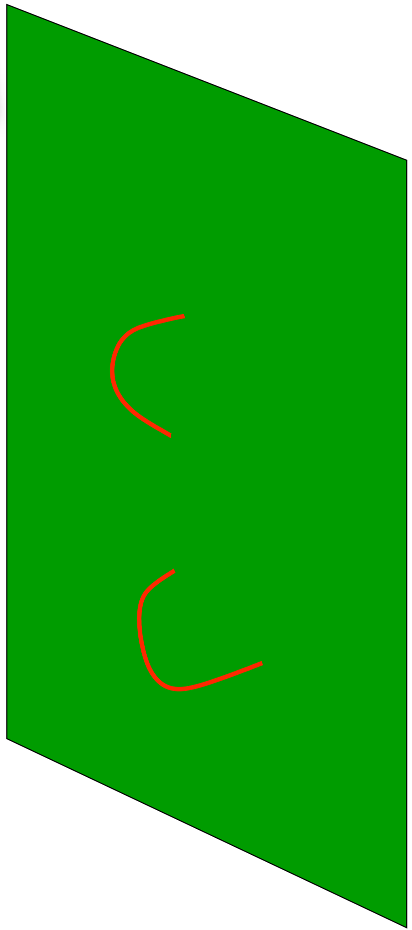
closed strings move in the extra dimensions!

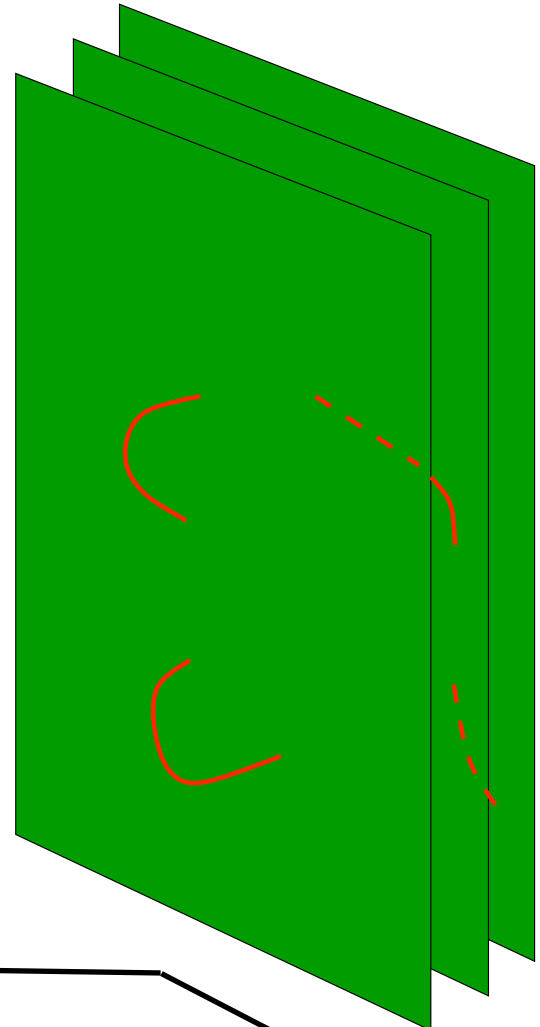
Key: Previous attempt was not using stringy ideas to address the right physical regime!

# It's More than Strings

The role of extended objects, "branes", is crucial in all this.

Consider flat space...





D3-brane is 3-dim hypersurface that moves in time. Strings end on it. Their physics captures aspects of its dynamics. The physics is U(1) susy gauge theory.

N of them gives SU(N)...

# It's More than Strings

The collective dynamics of the brane is just a low energy sector of the full string theory...

Brane also has a gravitational footprint. Decoupling the gauge theory from the rest of the stringy physics yields "near-horizon" geometry: $\mathrm{AdS}_5 \times S^5$

$$ds^2 = \frac{u^2}{R^2}( \quad dt^2 + d\vec{x} \;\; d\vec{x}) + \frac{R^2}{u^2}du^2 + R^2(d\theta^2 + \cos^2\theta d\Omega_3^2 + \sin^2\theta d\phi^2)$$

Reliable computations can be performed in the geometry if $R$ is large. This is why the gauge theory is at strong ('t Hooft) coupling...

$$R^2 = \sqrt{4\pi g_s N}\,\alpha' \;; \quad 2\pi g_s = g_{\mathrm{YM}}^2 \qquad\qquad N \text{ large; } g_s \text{ small; } \lambda = g_{\mathrm{YM}}^2 N \text{ large}$$

# The Full Duality

$\mathcal{N} = 4$ Supersymmetric $SU(N)$ Yang Mills

Superconformal invariance

$\sim SO(4,2)$

$A_\mu, 4\lambda, 6\varphi$ gauge multiplet

global R-symmetry $SU(4) \sim SO(6)$

isometries

$\mathrm{AdS}_5 \times S^5$

isometries

Review: Aharony, Gubser, Maldacena, Ooguri, Oz, hep-th/9905111

# More on the Dictionary

Field theory quantities read off using asymptotic behaviour of "bulk" fields.

$$Z_{\rm FT}(\partial M, \phi_{0,k}) = Z_{\rm grav}(M, \phi)$$

$$I_{\rm FT} \rightarrow I_{\rm FT} + \int_{\partial M} d^4y\, \phi_{0,k}(y) O_k(y)$$

Precise relation between masses of fields and dimensions of operators in theory...

# More on the Dictionary

Field theory quantities read off using asymptotic behaviour of "bulk" fields.

boundary at $z=0$

$$ds^2 = \frac{L^2}{z^2}(-dt^2 + dr^2 + r^2 d\Omega^2_{d-2}) + \frac{L^2}{z^2}dz^2$$

$$\phi(z, y^i) \to \phi_1(y^i) z^{\#_1} + \phi_2(y^i) z^{\#_2} + \cdots$$

Normalizable behaviour near bdry controls vev of operator

$$\Delta = \frac{1}{2}\left(d + \sqrt{d^2 + 4(mL)^2}\right)$$

Non-normalizable behaviour near bdry is an insertion of operator

$$m^2_{\rm BF} = -\frac{d^2}{4L^2}$$

# Recipe #2: Heavy Quarks

## Ingredients:

Five Spacetime Dimensions
Gravity
A negative cosmological constant
Strings that begin and end either at origin or infinity.

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}(R-2\Lambda)$$

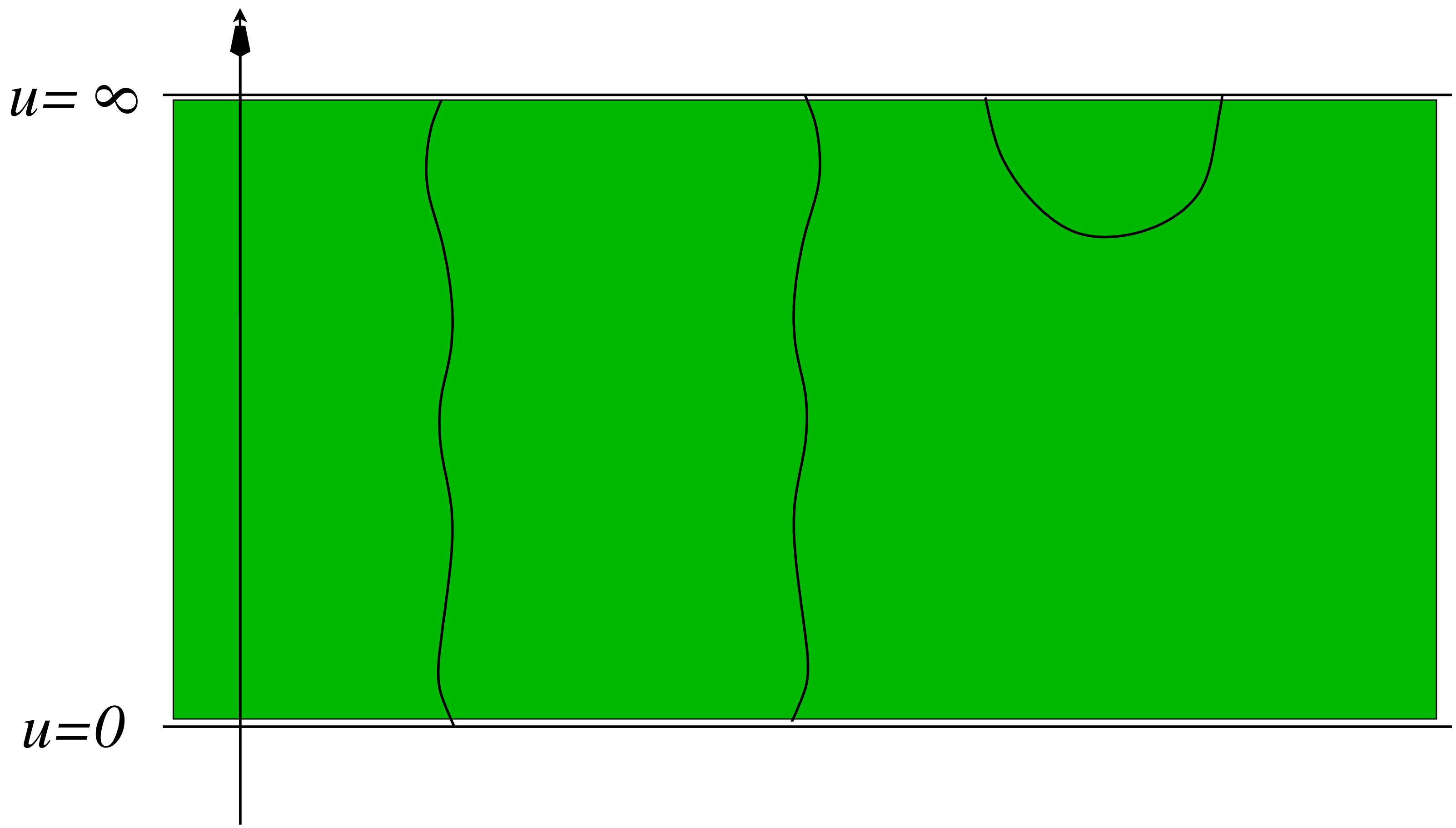
$$\Lambda = -\frac{6}{\ell^2}$$

## Method:

Arrange the strings according to those rules.

The ends of the strings are quarks.

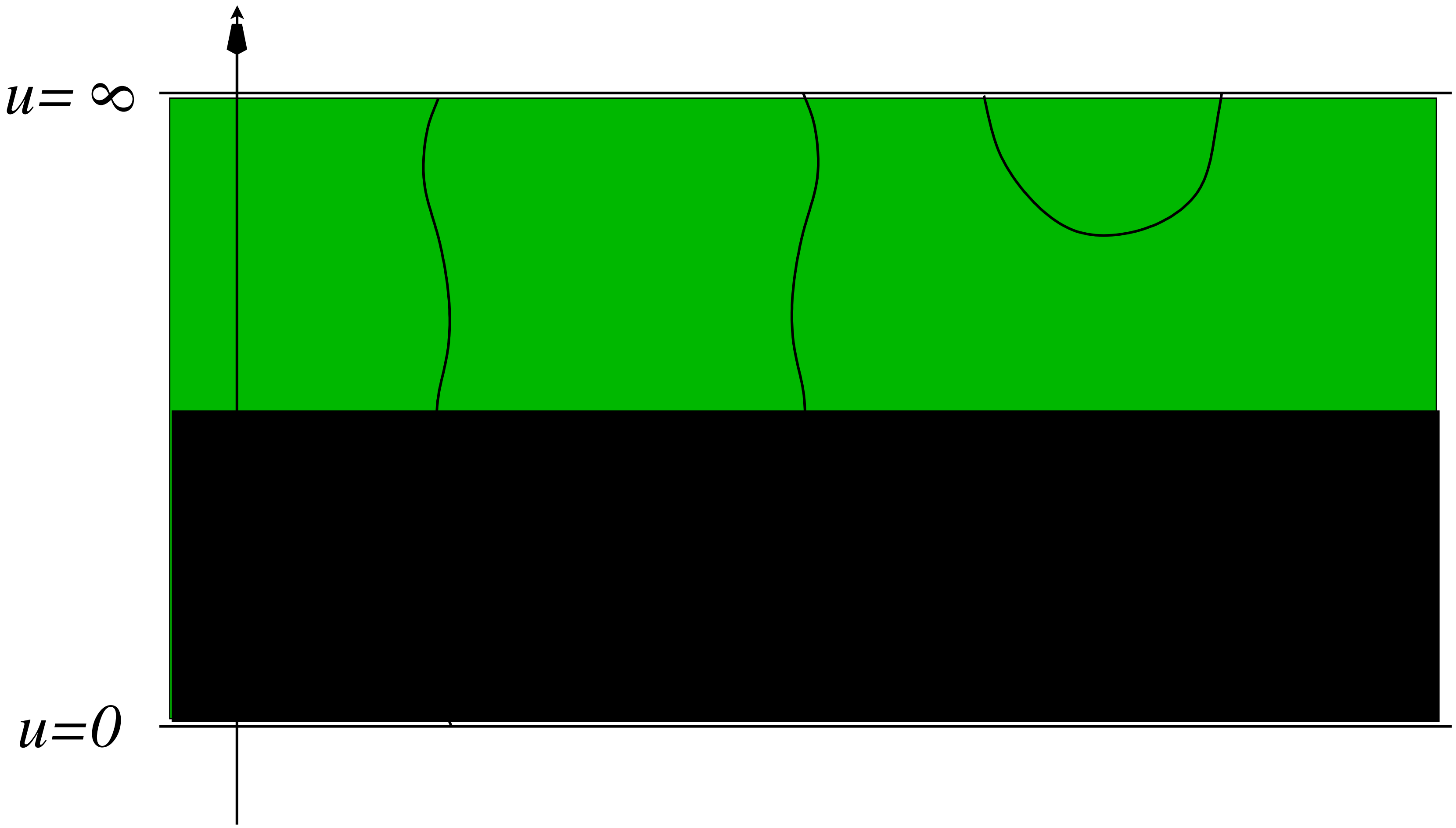
# Recipe #2: Heavy Quarks




Fundamental Quarks are infinitely heavy in this theory, since the strings are infinitely long...

Nevertheless, can study some aspects of heavy quark dymamics...

# Recipe #2: Heavy Quarks




Can study how quarks interact with the plasma by studying string in black hole background...

Reproduced aspects of jet-quenching this way...

Other recipes (and what can be learned from them) skipped due to lack of time:

## Recipe #3: Heavy Hadrons

How to make mesons and baryons

## Recipe #4: Fun with Charged Glue

Deconfining phase transitions with a toy model of baryon density

## Recipe #5: Adding Flavour

How to make dynamical quarks and mesons

## Recipe #6: Cooking the Mesons

Meson melting transitions and chiral symmetry breaking/restoration

# Hopes and Expectations

This is not QCD, but it is still remarkable what it can already do...

Can we hope for a real, controllable QCD dual?

Would need to:

* Find fully backreacted geometry for quarks
* Solve strings in highly curved backgrounds
* etc, etc..

May be as hard as solving QCD.

# Hopes and Expectations

The key issue (for me) is universality.

Let us not be too fixed on finding a QCD dual. Getting close might be enough...

What features of these simpler models persist to teach us about QCD?

String, gravity, etc, might be just the right variables for framing the universality question...

And black holes are just perfect for that, so finite temperature hydrodynamics might be in the best shape...

# Part II

## Chilling with the fermions:

## More Recipes... Best served cold

# Recipe #7: Transport in 2+1 D

## Ingredients:

Four Spacetime Dimensions
Gravity + Maxwell
A negative cosmological constant

## Method:

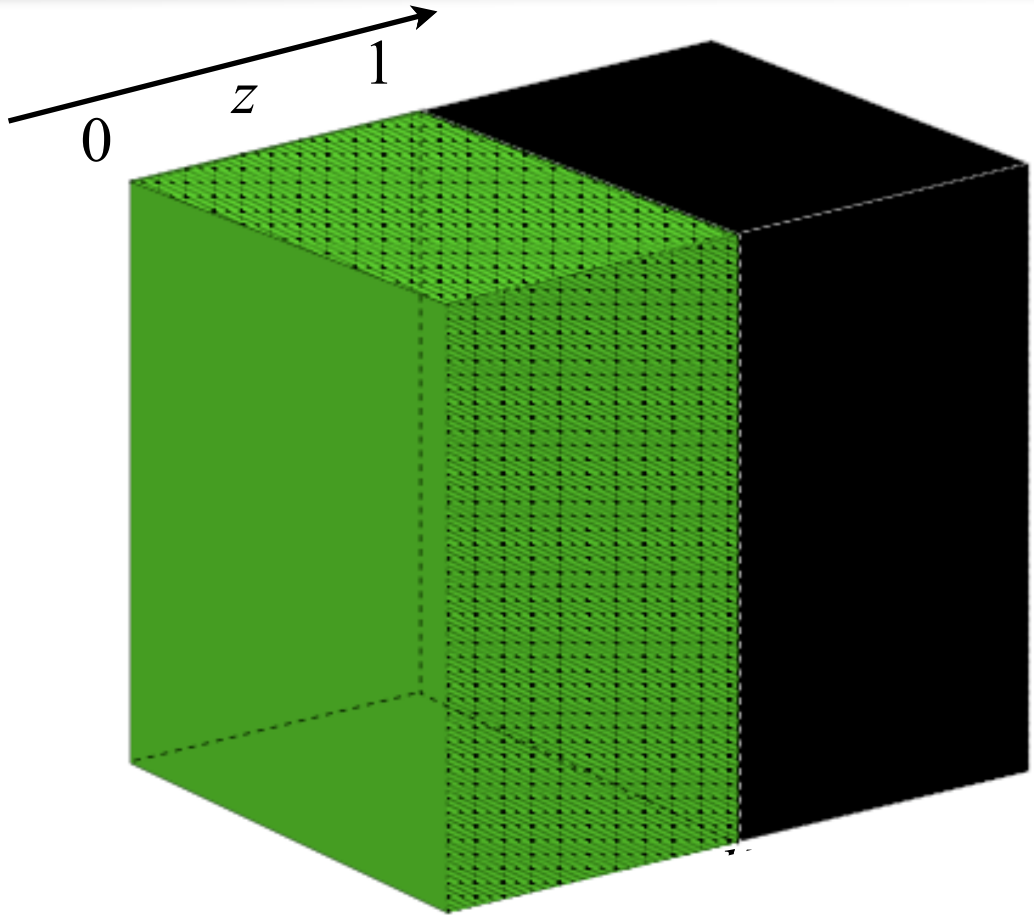
Place it all in a container that is asymptotically Minkowski on the boundary.

Pop in a (AdS) black hole

Why do this...?

# Recipe #7: Fun in 2+1 D

**At finite temperature:**



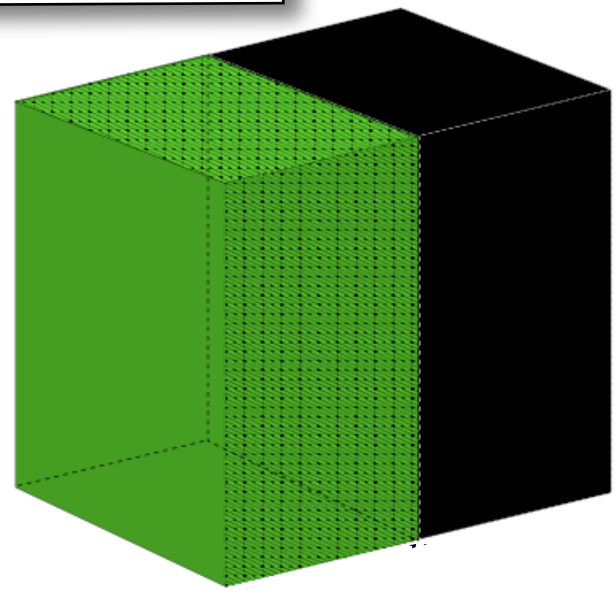

$$ds^2 = \frac{L^2\alpha^2}{z^2}\left(-f(z)\,dt^2 + dr^2 + r^2 d\phi^2\right) + \frac{L^2}{z^2}\frac{1}{f(z)}dz^2$$

$$f(z) = 1 - z^3$$

Black hole horizon at finite radius.

# Recipe #7: Fun in 2+1 D

What is this theory?



Another thermal theory, but now in 2+1 dimensions.

It is not Yang-Mills, but some thermalized version of an "exotic" fixed point theory defined by RG flow.

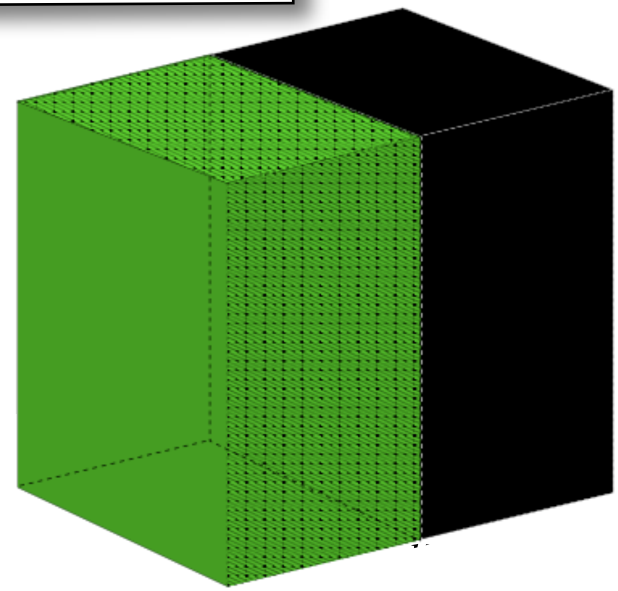
$$ds^2 = \frac{L^2\alpha^2}{z^2}\left(-f(z)\,dt^2 + dr^2 + r^2 d\phi^2\right) + \frac{L^2}{z^2}\frac{1}{f(z)}dz^2$$

$$f(z) = 1 - z^3$$

We can play with this...

# Recipe #7: Fun in 2+1 D

Now add electric and magnetic charge to the black hole.



$$ds^2 = \frac{L^2\alpha^2}{z^2}\left(-f(z)\,dt^2 + dr^2 + r^2 d\phi^2\right) + \frac{L^2}{z^2}\frac{1}{f(z)}dz^2$$

$$f(z) = 1 + \left(h^2 + q^2\right)z^4 - \left(1 + h^2 + q^2\right)z^3$$

$$A = h\alpha^2 r^2 d\phi + 2q\alpha\,(z-1)\,dt$$

$$T = \frac{1}{\beta} = \frac{\alpha}{4\pi}\left(3 - h^2 - q^2\right)$$

magnetic

electric

Meaning?

# Recall part of the Dictionary

Field theory quantities read off using asymptotic behaviour of "bulk" fields.

$$Z_{\rm FT}(\partial M, \phi_{0,k}) = Z_{\rm grav}(M, \phi)$$

$$I_{\rm FT} \to I_{\rm FT} + \int_{\partial M} d^4y\, \phi_{0,k}(y) O_k(y)$$

Precise relation between masses of fields and dimensions of operators in theory...

# Recall part of the Dictionary

Field theory quantities read off using asymptotic behaviour of “bulk” fields.

boundary at $z = 0$

$$ds^2 = \frac{L^2}{z^2}(-dt^2 + dr^2 + r^2 d\Omega^2_{d-2}) + \frac{L^2}{z^2} dz^2$$

$$\phi(z, y^i) \to \phi_1(y^i) z^{\#_1} + \phi_2(y^i) z^{\#_2} + \cdots$$

Normalizable behaviour near bdry controls vev of operator

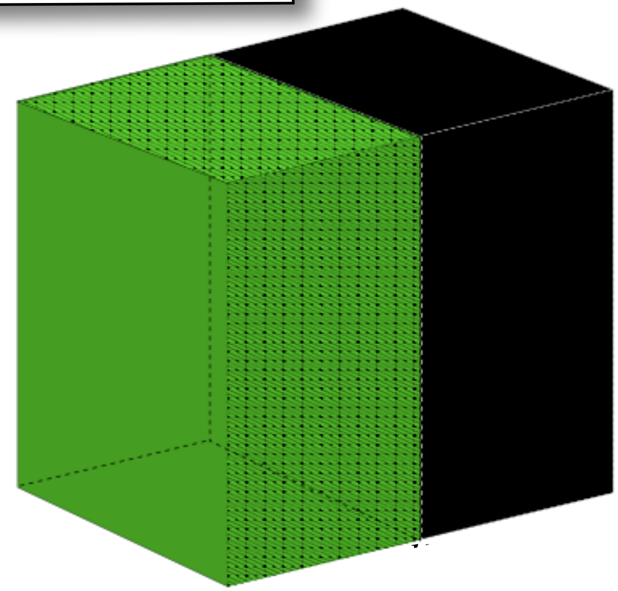
$$\Delta = \frac{1}{2}\left(d + \sqrt{d^2 + 4(mL)^2}\right)$$

Non-normalizable behaviour near bdry is an insertion of operator

$$m^2_{\mathrm{BF}} = -\frac{d^2}{4L^2}$$

# Recipe #7: Fun in 2+1 D

**Meaning for us?**



$$A_\phi(r) = h\alpha^2 r^2$$

$$\Rightarrow \quad B = \frac{1}{r}\partial_r A_\phi = 2h\alpha^2$$

external magnetic field

$$A_t = 2q\alpha z - 2q\alpha$$

density $\rho$

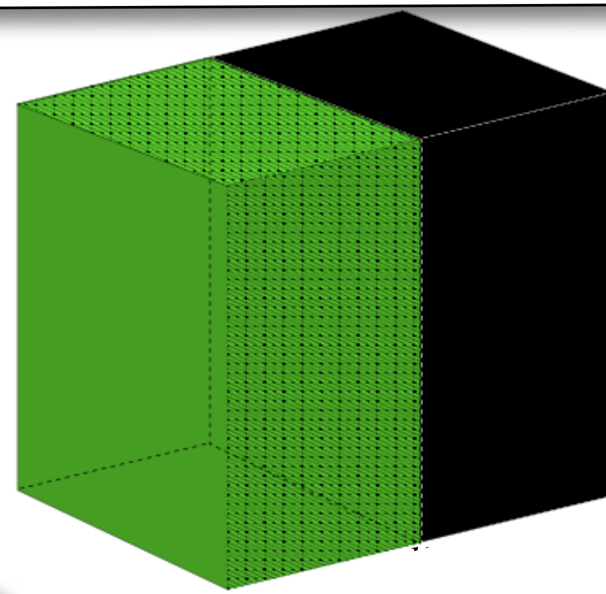
chemical potential $\mu$

Useful Laboratory for various things... e.g Hall effect, Nernst effect...

# Recipe #8: Superconductivity in 2+1 D

**Ingredients:**

Four Spacetime Dimensions
Gravity + Maxwell + charged scalar
A negative cosmological constant



**Method:**

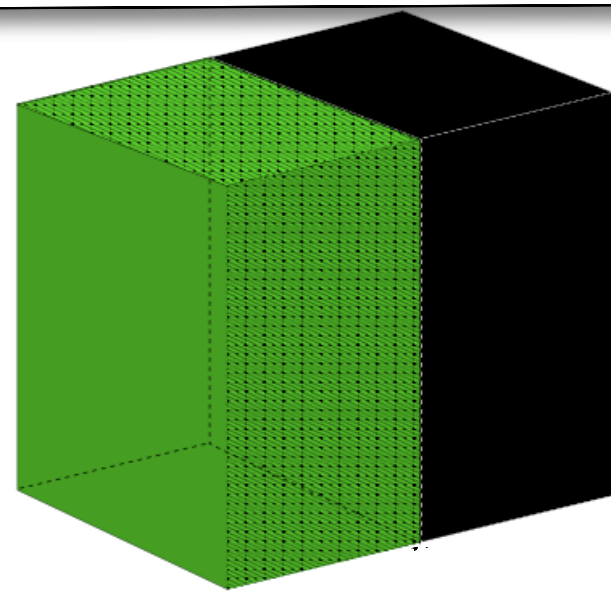
Place it all in a container that is asymptotically Minkowski on the boundary.

Put at finite temperature

Minimally couple the scalar and add a potential...

# Recipe #8: Superconductivity in 2+1 D

**What will happen?**



$$S_{\rm bulk} = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-G}\left\{R + \frac{6}{L^2} + L^2\left(-\frac{1}{4}F^2 - |\partial\Psi - igA\Psi|^2 - V(|\Psi|)\right)\right\}$$

**Don't panic!**

$$V(|\Psi|) = -\frac{2}{L^2}\bar{\Psi}\Psi$$

$$m_\Psi^2 = -\frac{2}{L^2} > m_{\rm BF}^2 = -\frac{9}{4L^2}$$

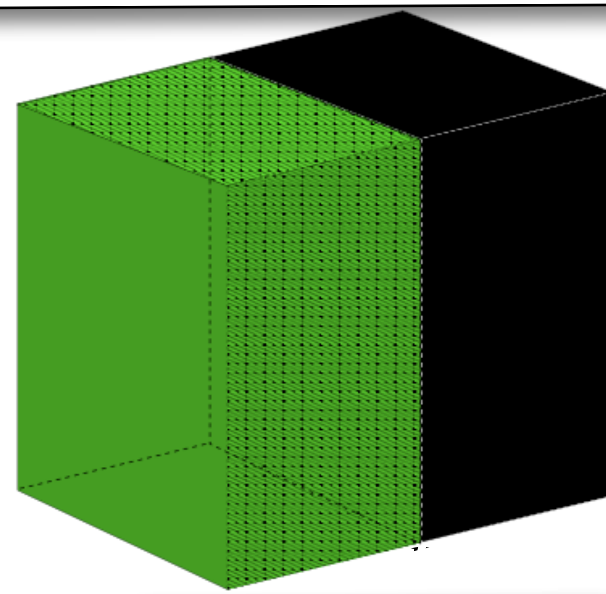
If B=0, can choose: $\Psi = \tilde{\rho}e^{i\theta}\ ;\theta = \text{const.}$

**We know we will have a black hole, but what will we get?**

# Recipe #8: Superconductivity in 2+1 D

What will happen?

$$T = \frac{\alpha}{4\pi}(3 - q^2)$$



High temperatures:

Reissner-Nordstrom Black Hole

$$\Psi = 0$$

But scalar mass is affected by $A_t$

Below a certain temp, it drops below BF bound!

Low temperatures:

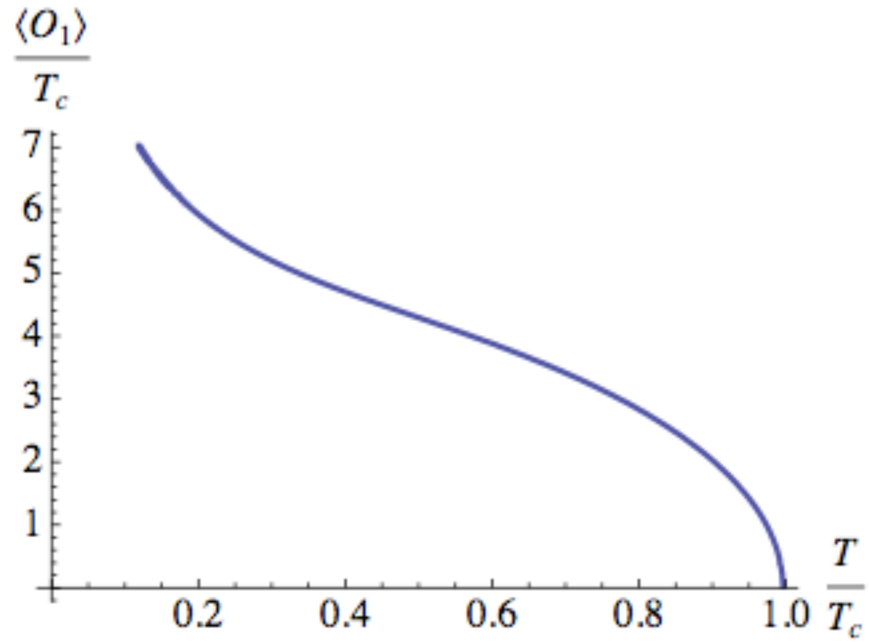
Black Hole with nontrivial charged scalar profile

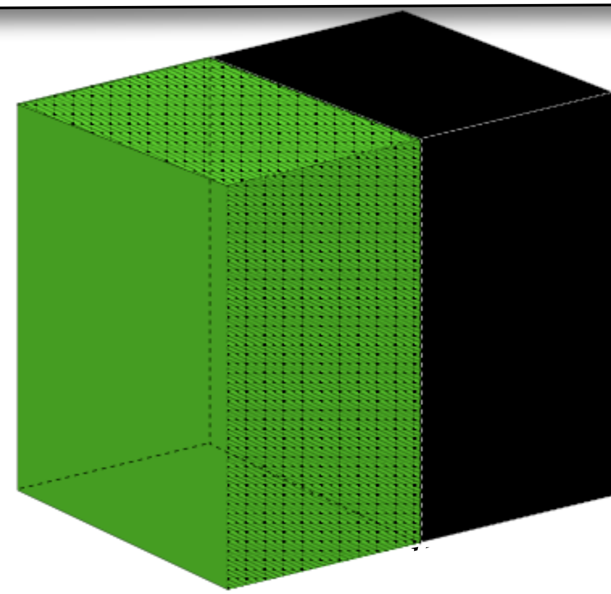
$$\tilde{\rho} \to \langle \mathcal{O} \rangle z + \cdots$$

Spontaneous breaking of the *U(1)*

# Recipe #8: Superconductivity in 2+1 D

Difficult to do exactly, but can do numerical work in various limits....



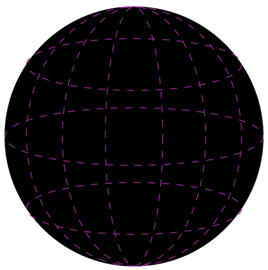



Can compute the DC conductivity and find it diverges... etc... etc..

Can go on to study system in presence of magnetic field...

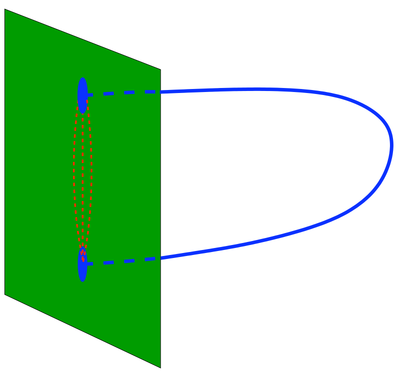
# Hopes and Expectations

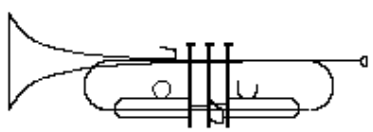
This is all very powerful use of the tools that come from string theory...



Real contact with experiment? Useful language? Already been useful at RHIC...

May feed back into other applications of string theory (unification, cosmology, etc...)





The work continues.
It is very exciting...!

# Some Reviews and guide to literature:

**General AdS/CFT technology:**

O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri, and Y. Oz, Phys. Rept. 323, 183 (2000), hep-th/9905111

**Applications to Heavy Ion collisions; Quark-Gluon plasma:**

M. Natsuume (2007), hep-ph/0701201

**Applications to Condensed Matter Physics:**

S. A. Hartnoll (2009), 0903.3246

C. P. Herzog (2009), 0904.1975