Teaser Question

Basic Training in Condensed–Matter Theory
Craig Fennie; Due Friday, April 3, 2009 (try to write something for the first one, but also try to think about the second one)

There was a brief mention in class that a ferroelectric is also ferroelastic. What we mean is that at the a second order ferroelectric phase transition, the system simultaneously undergoes a ferroelastic transition.

Show that Landau won’t be upset with such a statement. (Using group-theoretic arguments, one should be able to prove the above is always true but don’t worry about it for Friday.) This is what is meant when we say that $P$ is the primary order parameter, while $\eta$ is a secondary order parameter. Said another way that the system undergoes a proper ferroelectric transition and an improper ferroelastic transition.

Answer:

The electrical polarization is fundamental to the theory of insulating materials (and central to the theory of dielectrics and ferroelectrics) yet we didn’t know if $P$ was even a bulk quantity until the early 1990’s when David Vanderbilt (building on the work of Rafaelle Resta and others) devised the Berry phase theory of electrical polarization. The most surprising aspect of this theory is that $P$ has nothing to do with the charge density (i.e., the modulus of the wavefunction) but rather the phase of Bloch orbitals.

What is wrong with defining $P = \frac{1}{V} \int \rho(r) r \, dr$ for an extended system (i.e., Bloch system). Why is it ok for a finite system?
Answer: