HW Question

Basic Training in Condensed–Matter Theory
Craig Fennie; Due May, 2009 (talk with Erich)

The linear magnetoelectric effect vs. weak ferromagnetism.

Let's consider the real materials, Cr$_2$O$_3$ and Fe$_2$O$_3$. Both form in the Corundum (or if you want to sell it, “Sapphire”) crystal structure, Space group R̅3c. Attached shows a simplified view of the crystal structure and the relevant crystal symmetries. For each case:

- (a) What are the generators of the magnetic point group (take the generators of 3m and combine with time reversal symmetry, which operations leave the system invariant.)

- (b) Determine whether the system displays a non-zero linear magnetoelectric effect, weak ferromagnetism, or both (teaser: in general, can a system simultaneously display both the linear magnetoelectric effect and weak ferromagnetism? If so when)

- (c) for the system that displays a linear ME effect, what are the non-zero components of the ME tensor.)

- (d) write down the simplest Landau theory for each case (no higher than quadratic) for each system in terms of $P, L, M$.

Note, weak ferromagnetism can be thought of arising from terms like $E_{wFM} = D_{ij}L_iM_j$ (typically arising from a Dzyaloshinskii-Moria type of interaction $E_{wFM} = D \cdot (L \times M)$) whereas the linear magnetoelectric effect $E_{ME} = \gamma_{ijk}P_iL_jM_k$ where $D$ is an axial vector and $P$ is a polar vector.

A great source for to read up on this HW is Landau and Lifshitz, Electrodynamics of Continuous Media. Also E.A. Turov, “Can the magnetoelectric effect coexist with weak piezomagnetism and ferromagnetism,” Physics - Uspekhi 37 (3), 303 (1994).
Weak ferromagnetism vs ME effect

Space group: $\bar{R}\bar{3}c$ #167
Point group: $\bar{3}m$ (generators: $C_3$, $I$, $U_2$)

Spins are at Wyckoff position 3c: $(0,0,z)$ $(0,0,-z+\frac{1}{2})$ $(0,0,-z)$ $(0,0,z+\frac{1}{2})$