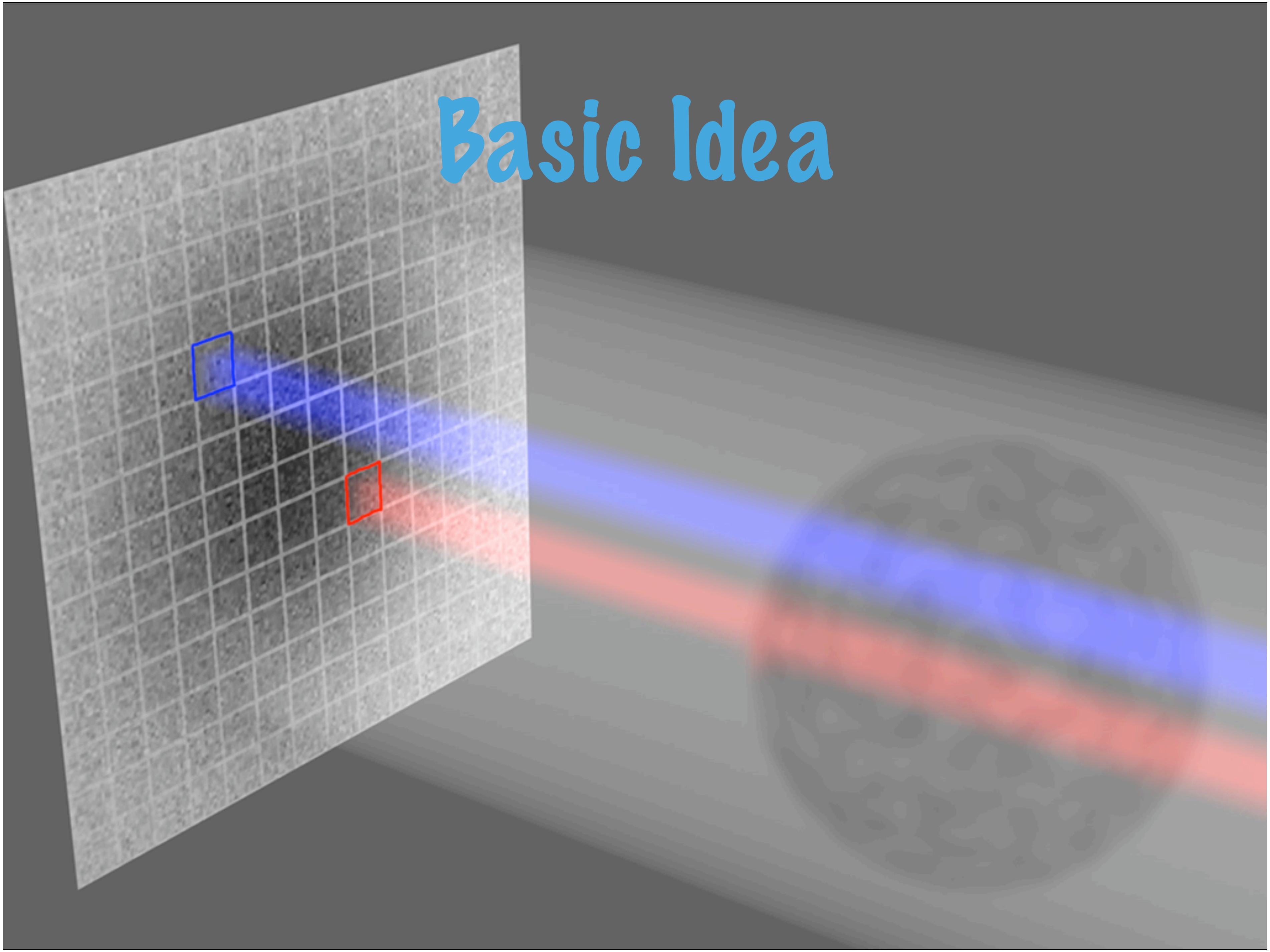
Absorption Imaging

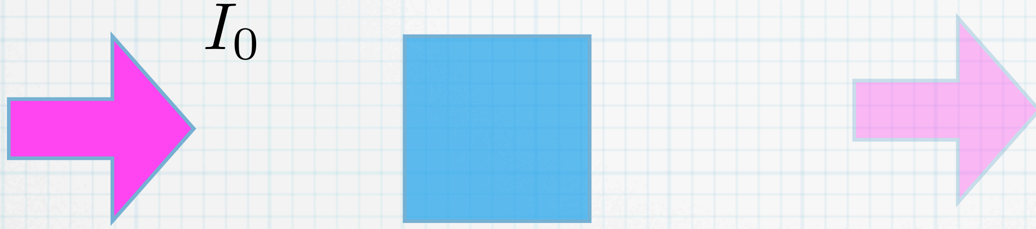
Dec 18, 2009

Basic Idea



Interpreting Images

Light: intensity I_0



Light: intensity I_f

gas of density n

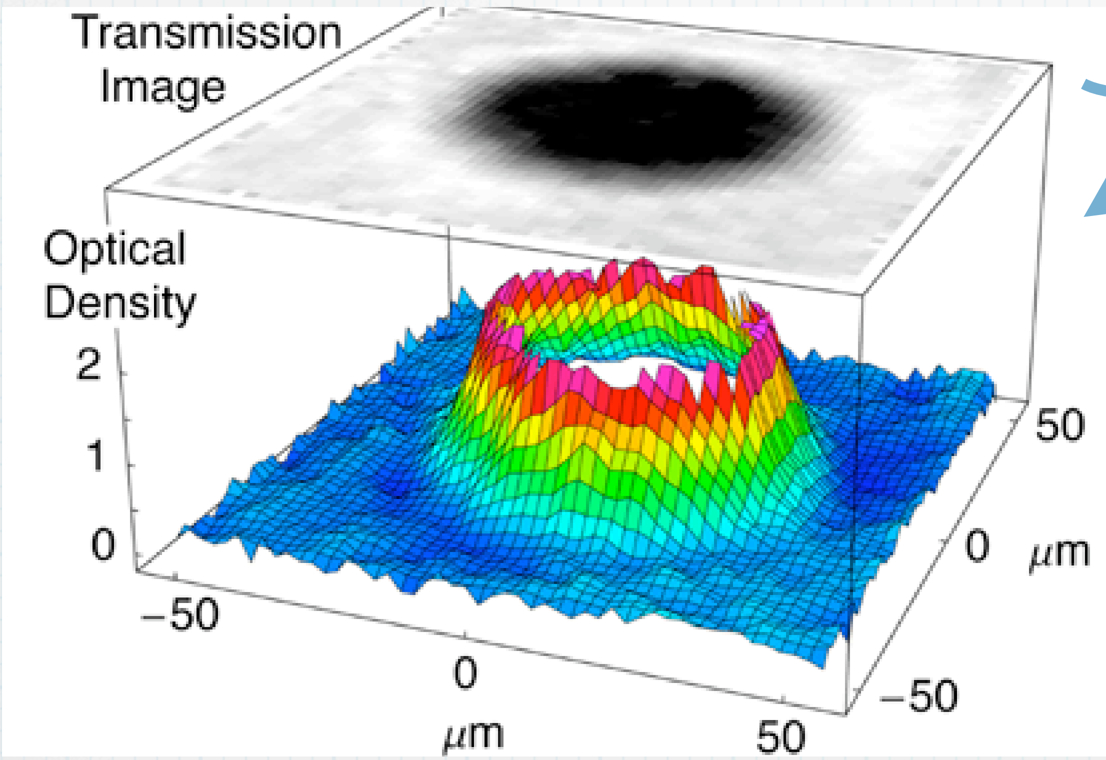
$$\frac{I_f}{I_0} \equiv e^{-\mathrm{O.D.}}$$

Defines Optical Density

$$\frac{dI}{dz} = -n\sigma I$$

$$\mathrm{O.D.} = \sigma \int n(z)dz$$

Example



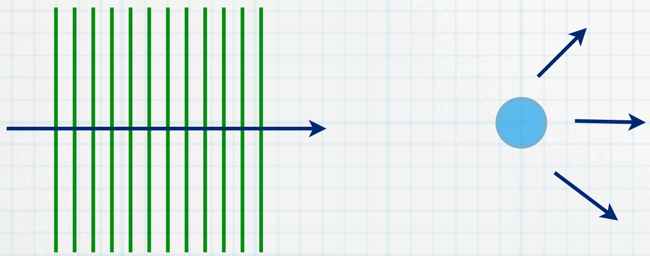
Take Log

$$\text{O.D.} = \sigma \int n(z)dz$$

Questions

* What can you learn from these images?
  * Simple answer: column density
  * Extract many things from this
* Where did the light go?

Where did the light go?



A: Scattered

Standard formalism:

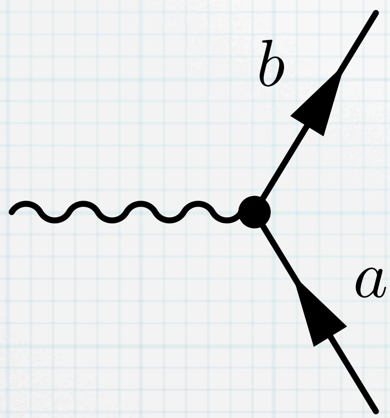
Classical EM field coupled to 2-level atom

(Assume you have all seen)

Here:

Imaginary part of photon self-energy

Photon-Atom interaction

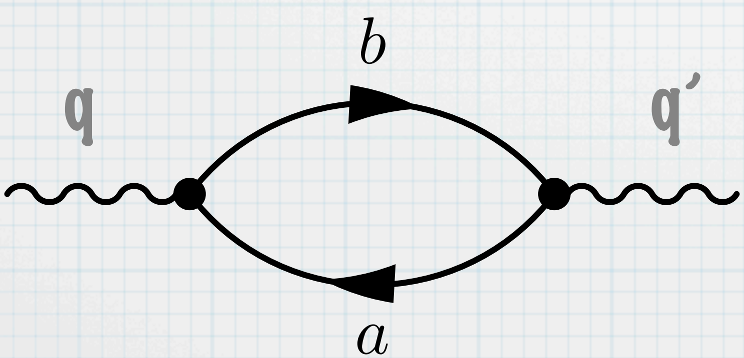


Neglect recoil

$$H = \sum_q \lambda b^\dagger a \alpha_q + \text{H.C.}$$

λ

dipole matrix element/Sqrt[V]

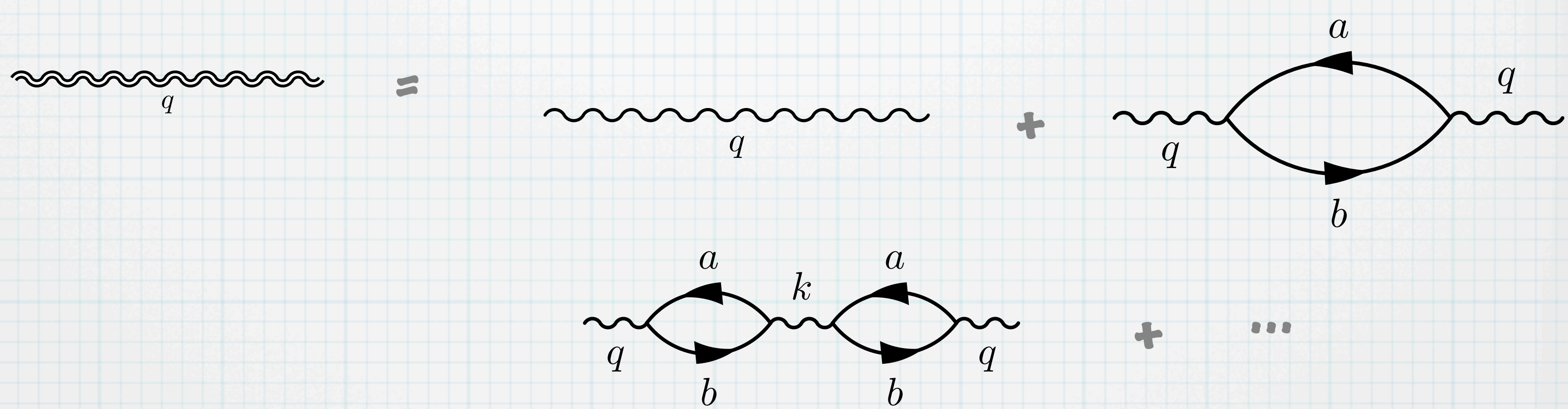
$\Sigma =$ 

$$\Sigma(q, q'; \omega) = \frac{\lambda^2}{\omega - \omega_0}$$

Is not diagonal

Photon Green Function

$$\left((\omega - ck)\delta_{kq} - \Sigma_{kq}\right) G_q = \delta_{kq}$$



$$G_{qq} = \frac{\frac{1}{\omega - cq}}{1 - \chi \frac{1}{\omega - cq}}$$

$$\chi = \frac{\frac{\lambda^2}{\omega - \omega_0}}{1 - \sum_{k'} \frac{1}{\omega - ck} \frac{\lambda^2}{\omega - \omega_0}}$$

Result

$$G_{qq} = \frac{1}{\omega - cq - \frac{\lambda^2}{\omega - \omega_0 - i\Gamma}}$$

$$\Gamma = \pi\lambda^2 \sum_k \delta(\omega - ck)$$

Get same result if you just couple to 2-level atom and let upper level have finite lifetime

$$G_{qq} \approx \frac{Z}{\omega - cq - \frac{\lambda^2}{cq - \omega_0 - i\Gamma}} + \text{incoherent}$$

Photon decay rate: $\dfrac{\lambda^2\Gamma}{(cq-\omega)^2+\Gamma^2} \equiv \dfrac{\sigma c}{V}$

Maximum cross-section

On resonance: $$\frac{1}{\sigma} \sim \frac{c\Gamma}{\lambda^2 V} \sim k^2$$

Optical Theorem:

You can't scatter more than comes in

$$e^{ikz} = \frac{e^{-ikr}}{2ikr} + \cdots$$

Total s-wave flux:

$$\Phi_s = \frac{c}{V}\frac{4\pi r^2}{4k^2 r^2} \geq \frac{c\sigma}{V}$$

$$\sigma \mathrm{max} = \frac{\pi}{k^2}$$

Intermission

Interpreting column densities

required signal to noise

Strategy 1:

Model trap and equation of state
- calculate column density
- compare with image

Strategy 2:

Model trap and extract EOS independent quantities
- virial theorem

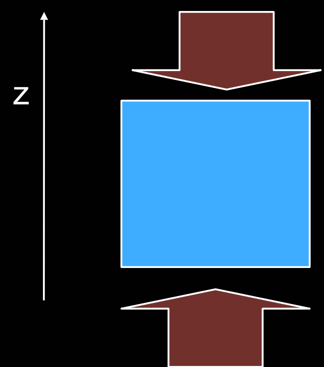
Strategy 3:

Model trap and extract EOS

Hydrostatics

Assume locally Homogeneous

$$F(z+\delta z) = P(z+\delta z) \times A$$



$$F(z) = P(z) \times A$$

$$F(z) - F(z+\delta z) = \frac{dV}{dz}\,\rho\, A\, \delta z$$

gives

$$\frac{dP}{dz} = -\rho \frac{dV}{dz}$$

Isothermal Assumption:

$$\frac{dP}{dz} = \rho \frac{d\mu}{dz}$$

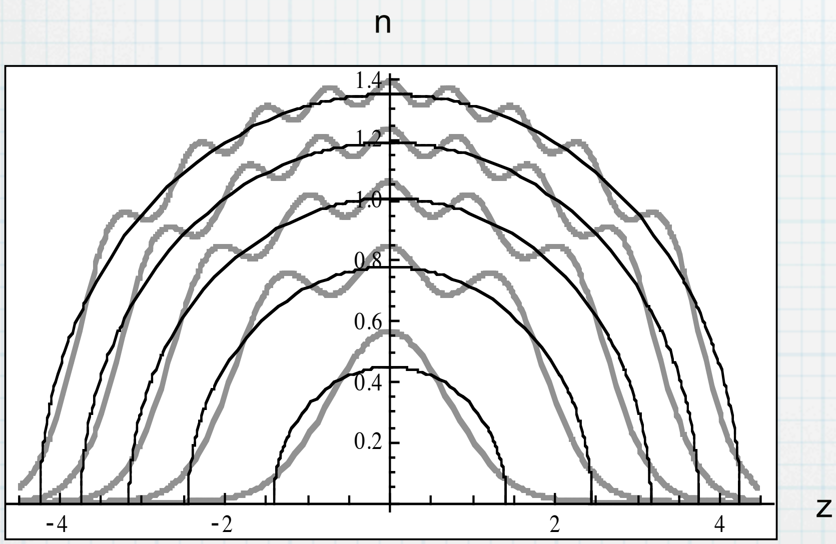
Result: Thomas-Fermi

$$\mu(z) = \mu_0 - V(z)$$

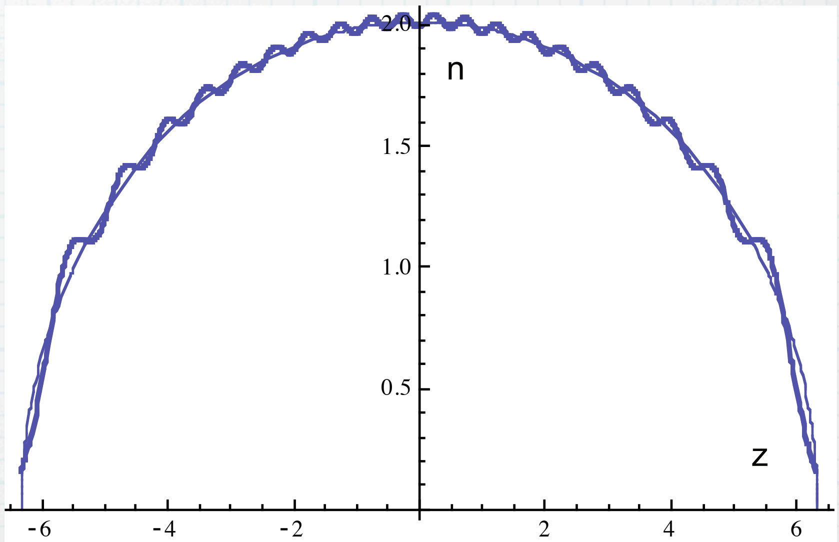
Punchline:
local properties of inhomogeneous system given by homogeneous equation of state

How good is assumption?

Non-interacting fermions in 1D -- very small particle numbers



N=20



Wiggles -- discreteness of particles

Scale as 1/N

Phase boundaries?

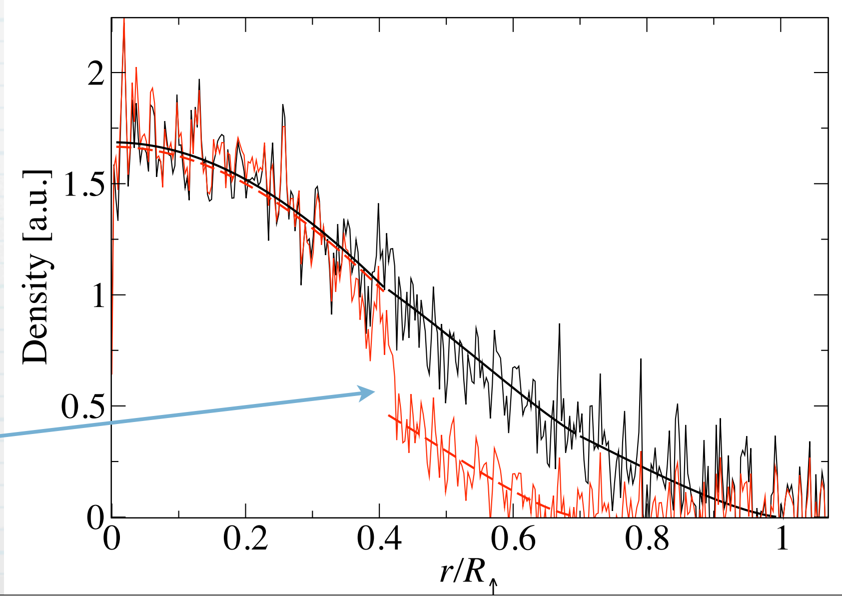
First order: good to thickness of domain wall (also need to model surface tension)

Second order: Good to scale where: $$\frac{1}{\xi} \sim \frac{V}{V'}$$

example: first order

black: up
red: down

1st order transition



Extracting density from column density

(not practical -- signal to noise)

$$dP = n d\mu + s dT$$

$$\mu(z) = \mu_0 - V(z)$$

Assume Harmonic trap

$$\begin{aligned} n_a(z) &= \int d^2 r_\perp n(\mu_c(z) - V_\perp(r_\perp)) \\ &= \frac{2\pi w^4 m}{\hbar^2 \lambda^2} \int_{-\infty}^{\mu_c(z)} d\mu [\lambda^2 n(\mu)] \\ &= \frac{2\pi w^4 m}{\hbar^2 \lambda^2} P_c(z), \end{aligned}$$

$$\begin{aligned} \frac{\partial n_a(z)}{\partial z} &= \int d^2 r_\perp \frac{\partial n(\mu_c(z) - V_\perp(r_\perp))}{\partial \mu} \left(- \frac{2\hbar^2 z}{m d^4} \right) \\ &= - \frac{8\pi z w^4}{d^4} \int_{-\infty}^{\mu_c(z)} \frac{\partial n}{\partial \mu} \\ &= - \frac{8\pi z w^4 n_c(z)}{d^4 \lambda^2}, \end{aligned}$$

Integrate to get "Axial Density"

Differentiate

Abel Transform

Assume cylindrical symmetry

$$n(r_\perp, z) \xrightarrow{\text{Abel}} n_c(x, z) = \int dy\, n(\sqrt{x^2 + y^2}, z)$$

$$= 2 \int_x^\infty \frac{r\,dr}{\sqrt{r^2 - x^2}}\, n(r, z)$$

Inverse Abel

$$n(r, z) = -\frac{1}{\pi} \int_r^\infty \frac{dx}{\sqrt{x^2 - r^2}} \frac{dn_c(x, z)}{dx}$$

Problem: Adds noise

Generic Problem

Given: Data set is convolved

$$F(\mathbf{r}) = \int d\mathbf{s} K(\mathbf{r},\mathbf{s}) f(\mathbf{s})$$

Measured

Known

Desired

Example 1: K is aperture function of optics

Example 2: K is projection from 3D to 2D

Given: Data is noisy

(And inverse transform increases noise)

How to extract f?

Idea: Model Noise

Model Space
(ex 3D)

Data Space

Data

$$\{f_i\} \xrightarrow{K} \{F_\nu\} \qquad \{F_\nu^d\}$$

Model noise

$$F_\nu^d = \bar{F}_\nu^d + \sigma_\nu \xi_\nu$$

Ex: Independent Gaussian random variables with mean 0 and standard deviation 1

Compare model and data

$$\chi^2 = \sum_\nu \frac{(F_\nu - F_\nu^d)^2}{\sigma^2}$$

Want to choose f so that

$$F_\nu = \bar{F}_\nu^d$$

Idea: Model Noise

$$\chi^2 = \sum_\nu \frac{(F_\nu - F_\nu^d)^2}{\sigma^2}$$

If $\quad F_\nu = \bar{F}_\nu^d$

Most probable value of $\quad \chi^2$

$$\bar{\chi^2} = N$$

= number of pixels

Consider space of all f's which give this chi^2

Data

$$\{F_\nu^d\}$$

Model noise

$$F_\nu^d = \bar{F}_\nu^d + \sigma_\nu \xi_\nu$$

Ex: Independent Gaussian random variables with mean 0 and standard deviation 1

Maximum Entropy

Bayesian Principle:

If you don't know anything, assume everything is equally likely

Choose the f's which satisfy $\chi^2 = N$

and carry the least information

Maximize $S = -\sum_i f_i \log(f_i/M)$ $\qquad M = \sum_i f_i$