P683 HW2

Due Wednesay Feb 3, 2010

Problem 1. Typical Spectral Density One is typically interested in A, but can most easily calculate G. Here we will imagine we did a calculation of G, and want to know what A looks like. Let us take

$$G(\omega) = \frac{1}{\omega - \epsilon - \lambda\sqrt{-\omega}/2},$$

where ϵ and λ are real. We will derive this G in a future class. It is typical of an atomic state. By $\sqrt{x}$ we mean the principle brancy of the square root. Thus if $\mathrm{Im}(\omega) > 0$ and $\mathrm{Re}(\omega) > 0$, then $\mathrm{Im}(\sqrt{-\omega}) > 0$.

Define $F_\delta(\omega) = i[G(\omega + i\delta) - G(\omega - i\delta)]$, so that $A(\omega) = \lim_{\delta\to 0} F_\delta(\omega)$.

1.1. Plot $F_\delta(\omega)$ for $\delta = 0.0001, \lambda = 0.02, \epsilon = 1$, over the range $-1 < \omega < 2$. Take the vertical axis to go from 0 to 200. Play a bit with the parameters. What happens as one takes $\delta \to 0$?

1.2. Plot $F_\delta(\omega)$ for $\delta = 0.0001, \lambda = 4, \epsilon = -1$, over the range $-1 < \omega < 2$. Take the vertical axis to go from 0 to 2. Play a bit with the parameters. What happens as one takes $\delta \to 0$? What happens as one makes λ bigger? You will need to adjust the vertical scale.

1.3. (bonus) Analytically calculate A. Its not that hard – but the expression is not as revealing as making the previous plots.

Problem 2. Analytic Structure of G The greens function is related to the spectral density by

$$G(\omega) = \int \frac{dz}{2\pi} \frac{A(z)}{\omega - z}.$$

2.1. Let $A(z) = 2\pi\delta(z - \epsilon)$, where ϵ is real. What is G? Is it analytic away from the real axis?

[Note, since any A can be written as some limit of delta-functions, this immediately gives us a "physicist proof" of the analyticity of G away from the real axis.]

2.2. Suppose

$$A(z) = \frac{\Gamma}{(z - \epsilon)^2 + (\Gamma/2)^2}$$

with real ϵ and Γ. What is $G(\omega)$. Note G is discontinuous across the real ω axis, so one has to separately consider the case $\mathrm{Im}(\omega) > 0$ and $\mathrm{Im}(\omega) < 0$

Problem 3. By using the definition $A(\omega) = G^>(\omega) \mp G^<(\omega)$, show that $A(\omega)$ is real [for real ω].