P683 HW4

Due Wednesday Feb 10, 2010

Problem 1. Lipman-Schwinger equation

As a practice with the equation of motion approach, consider the Hamiltonian

$$H = \sum_k a_k^\dagger a_k + \sum_{kq} a_k^\dagger a_q.$$

Write down the equations of motion for $a_k(t)$, and use them to derive an equation obeyed by $G_{kq}(t) = (1/i)\langle T a_k(t) a_q^\dagger(0)\rangle$. Fourier transform with respect to time to arrive at the Lipman-Schwinger equation.

Problem 2. Analytic Structure of the Self-Energy In a few lectures we will encounter a self-energy of the form

$$\begin{aligned}\Sigma(p, t-t') \quad &= \quad \pm i^2 \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3}\frac{1}{2}\left(V(p-p_2) \pm V(p-p_3)\right)^2 (2\pi)^3\delta(p+p_1-p_2-p_3) \\ &\qquad \times G(p_1, t'-t)G(p_2, t-t')G(p_3, t-t').\end{aligned}$$

This self-energy may be written as

$$\Sigma(p, t-t') = \frac{1}{i}\theta(t-t')\Sigma^>(p, t-t') \pm \frac{1}{i}\theta(t'-t)\Sigma^<(p, t-p').$$

2.1. Write $\Sigma^>(p, t-t')$ and $\Sigma^<(p, t-t')$ in terms of $G^>$ and $G^<$.

2.2. Following the arguments given in class we should have

$$\Sigma(p,\omega) = \int \frac{dz}{2\pi}\frac{1}{\omega - z}\Gamma(p, z)$$

How is $\Gamma(p, z)$ related to $\Sigma^>$ and $\Sigma^<$.

2.3. Using the fact that $G^>(p,\omega) = (1 \pm f(\omega))A_p(\omega)$ and $G^<(p,\omega) = f(\omega)A_p(\omega)$, and the result of 2.2, write an expression of $\Sigma(p,\omega)$ in terms of the occupations f and the spectral function A. Do not do any integrals.

2.4. Can you write an expression for $\Gamma(p,\omega)$ in terms of the discontinuity of Σ accross the real axis? What is that expression?