P683 HW6

Due Friday Feb 19, 2010

Problem 1 (Applications of the Quantum Boltzmann Equation – a quantum wire)**.** Consider a wire of length L. Its left end is connected to a zero temperature particle bath with chemical potential μ_L, while the right hand end is connected to one with chemical potential μ_R. We will take $\mu_L > \mu_R$. The particles do not interact with one another, but they can elastically scatter off impurities in the wire. The kinematics in the wire are purely one-dimensional, so the only thing the impurities can do is scatter a particle with momentum p to $-p$ and vice versa. Throughout we will take $p > 0$ and separately consider "right movers" described by distribution $f_p(x)$ and "left movers" described by $f_{-p}(x)$. The Boltzmann equation for this situation reads:

$$\begin{aligned}\left[\partial_t + \frac{p}{m}\partial_x\right] f_p(x,t) &= \gamma\left[f_{-p}(1-f_p) - f_p(1-f_{-p})\right] \\ \left[\partial_t - \frac{p}{m}\partial_x\right] f_{-p}(x,t) &= -\gamma\left[f_{-p}(1-f_p) - f_p(1-f_{-p})\right]\end{aligned}$$

where all f's on the right are evaluated at position x and time t. The boundary conditions read

$$\begin{aligned}f_p(x=0,t) &= \theta(\mu_L - \epsilon_p) \\ f_{-p}(x=L,t) &= \theta(\mu_R - \epsilon_p).\end{aligned}$$

1.1. When $\gamma = 0$, find the steady state solution $f_{\pm p}(x)$ with $\partial_t f_{\pm p}(x) = 0$.

1.2. Show that when γ is large, then f_p obeys a diffusion equation.

Hint: Fourier transform the Boltzmann equation, and eliminate f_p. Neglect the ω^2 term compared to $\gamma\omega$.

1.3. Find the steady state $f_p(x)$ and $f_{-p}(x)$.

Hint: The easiest way to do this is to simply guess that $f_{\pm p}(x)$ is linear in x, then solve for the coefficients.