P683 HW1

Due Friday Jan 29, 2010

Problem 1. Ideal Gas Consider an ideal gas with Hamiltonian

$$H = \sum_k \epsilon_k a_k^\dagger a_k$$

where $\epsilon_k = k^2/2m - \mu$.

1.1. Use the Heisenberg's equations of motion for a_k to calculate $a_k(t)$ in terms of $a_k(0)$. Do this for both Bosons and Fermions.

Solution 1.1. Heisenberg equations of motion read

$$\begin{aligned} i\partial_t a_k &= [a_k, H] \\ &= \epsilon_k a_k, \end{aligned}$$

so $a_k(t) = a_k(0)e^{-i\epsilon_k t}$. The same argument yields $a_k^\dagger(t) = a_k^\dagger(0)e^{i\epsilon_k t}$.

1.2. Write an explicit expression for

$$\begin{aligned} G_k^>(\omega) &= \int dt\, e^{i\omega t} \langle a_k(t) a_k^\dagger(0)\rangle \\ G_k^<(\omega) &= \int dt\, e^{i\omega t} \langle a_k^\dagger(0) a_k(t)\rangle \end{aligned}$$

in terms of $n_k = \langle a_k^\dagger(0) a_k(0)\rangle$. You should use that $\int dt\, e^{i\nu t} = 2\pi\delta(\nu)$.

1.3. In class we showed that in equilibrium $G_k^>(\omega) = e^{\beta\omega} G_k^<(\omega)$. Use this to find n_k.

1.4. Find $A_k(\omega) = G_k^>(\omega) - G_k^<(\omega)$