CONFORMAL SYMMETRY AND APPLICATIONS
Condensed matter basic training, Spring 2013, André LeClair

1 Outline

GENERALITIES IN ANY DIMENSION

- Quantum mechanics in d spatial dimensions verses Classical statistical mechanics in $d+1$ dimensions.
- Scale invariance verses conformal invariance.
- Renormalization group[1]
- Minkowski verses Euclidean space.
- Dirac fermions near the Fermi surface on a lattice.
- Dirac fermions in topological insulators[2, 3, 4, 5].
- Majorana fermions[6].

FREE QUANTUM FIELD THEORY AS A GAUSSIAN PATH INTEGRAL

- Path integrals for quantum fields. Wick's theorem.
- Free fermions and bosons[7, 8].
- Bosonization.
- U(1) currents.
- U(1) Kac-Moody algebra.

LUTTINGER LIQUID

- Solution using Bosonization.
- A non-Fermi liquid.
- Fractional Quantum Hall effect edge states as a chiral Luttinger liquid[9].

FORMALISM[11, 7, 8]

- **Stress tensor, infinite conformal symmetry, Virasoro algebra.**
- **Virasoro central charge and finite size effects.**
- **Primary fields and anomalous dimensions.**
- **Minimal models summary.**
- **Coulomb gas techniques.**

EXAMPLES

- **Ising model as a Majorana fermion. Spin field and non-abelian statistics.**
- **O(N) models.**

CURRENT ALGEBRAS[12]

- **Affine Lie algebras.**
- **Sugawara construction.**
- **SU(2) case.**
- **Heisenberg spin chains.**
- **Cosets and minimal models.**

OFF-CRITICAL PERTURBATIONS

- **Renormalization group.**
- **Hubbard model and spin-charge separation[10].**
- **Integrable perturbations[14, 15].**
- **Off-critical Ising model in a magnetic field and E_8.**

References

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