P7654 HW4

Due Friday Feb 8, 2013

Problem 1. Spontaneous Emission We are going to consider the prototypical example of a dissipation:

$$H = \epsilon_b b^\dagger b + \sum_j \epsilon_j a_j^\dagger a_j + \sum_j \lambda_j (b^\dagger a_j + a_j^\dagger b), \tag{1}$$

This is an abstract model, where there is an isolated mode of energy ϵ_b coupled linearly to a large number of other modes, each with energy ϵ_j.

Some concrete realizations include, for example, an impurity in metal, or the spontaneous emission of an excited atom: ϵ_b could the energy of the excited atom, and ϵ_j is the energy of the ground state atom plus a single photon in mode j. It turns out the answer here will not depend upon if we take these operators to be Bosonic or Fermionic. Use whichever you prefer.

We are going to calculate the Greens function

$$G_{bb}(t) = \frac{1}{i}\langle T b(t) b^\dagger(0)\rangle.$$

1.1. Using the techniques we have developed in this class, write $G_{bb}(t)$ as a perturbation series in λ.

1.2. Sum the series to get the Fourier transform $G_{bb}(\omega)$.

One defines the "self-energy" via the expression

$$G(\omega) = \frac{1}{\omega - \epsilon_b - \Sigma(\omega)}$$

1.3. Express $\Sigma(\omega)$ as a sum over the modes of the system.

Lets specialize to the case where the density of modes in the continuum is uniform:

$$\sum_j \frac{\lambda_j^2}{\omega - \epsilon_j} \to \lambda^2 \int \frac{d\nu}{2\pi} \frac{1}{\omega - \nu}$$

1.4. What is $\Sigma(\omega)$ in this case? (Assume the imaginary part of ω is non-zero. Consider both the case where this imaginary part is positive or negative.)

1.5. Calculate $G_{bb}^R(t)$.

1.6. Argue how your result relates to an experiment where at time $t = 0$ you have the b mode occupied, and all others unoccupied. How is $G_b^R b(t)$ related to the probability the mode b is occupied after time t?

1.7. How does this result compare to Fermi's Golden Rule?

As we will explore in more detail in the next problem set, generically the imaginary part of the self-energy gives the "decay rate" or the "scattering rate."