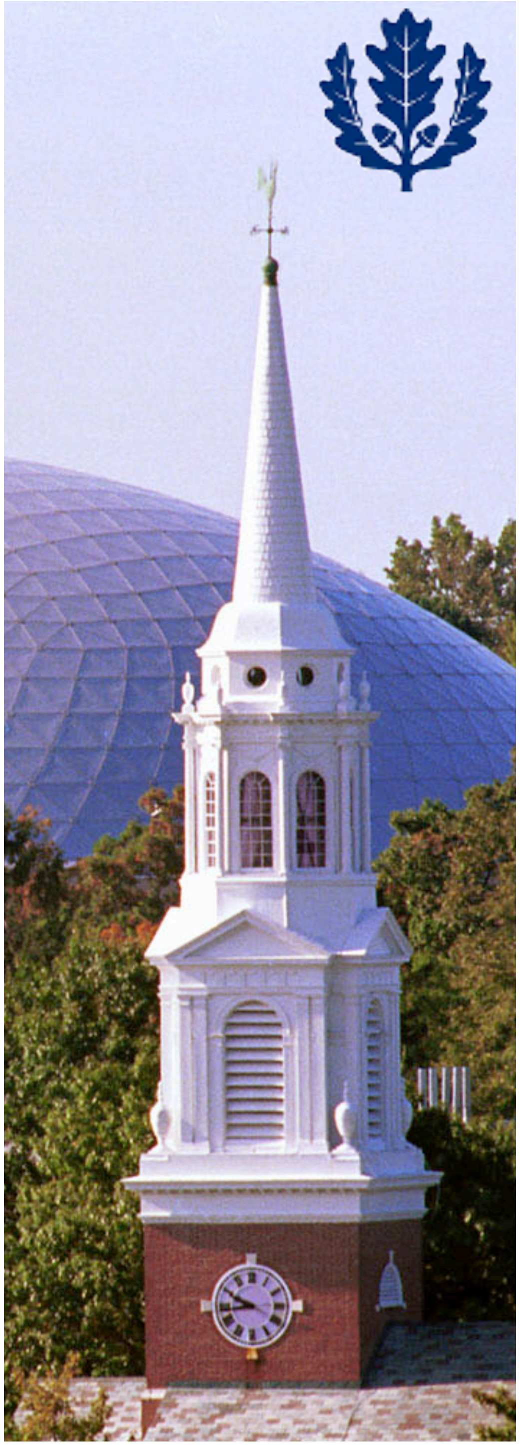


University of Connecticut

Aspen Center for PHYSICS

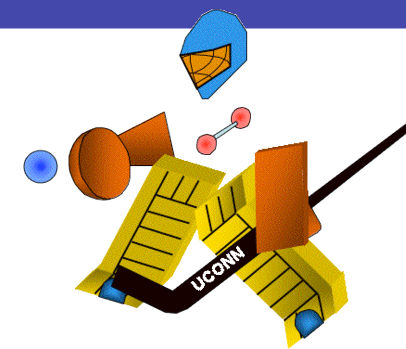
Workshop on
Quantum Simulation/Computation
with Cold Atoms and Molecules

AMO Tutorial 1: trapped ions

by Robin Côté

Aspen, Wednesday June 3 2009

Outline



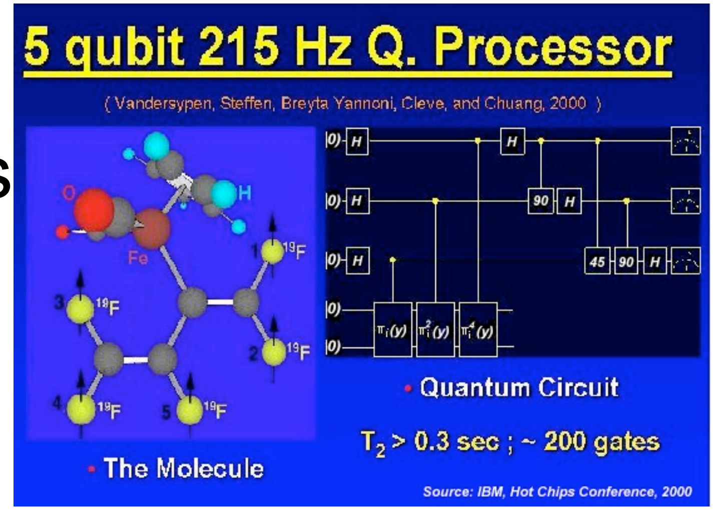
- Introduction/motivation
  - Property wish list
- Overview of AMO platforms
- Trapped ion / atom-ion systems
- Cold atoms
- Cold molecules
- Other applications
- Concluding remarks

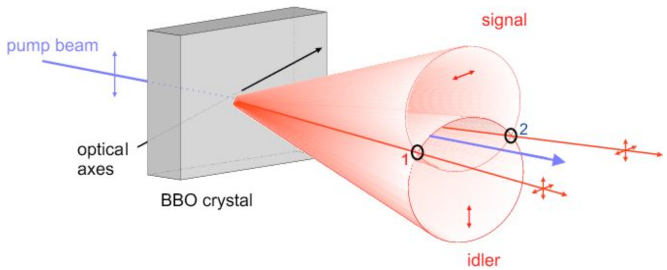
Quantum computer wish list

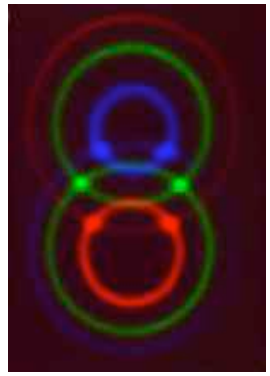
- Stable trapping/storage
- Addressable qubits
- Gates between (separated) qubits
- Long coherence-, short interaction-times
- Scalability
- Strong, switchable interaction
- Readout/initialization

Various platforms

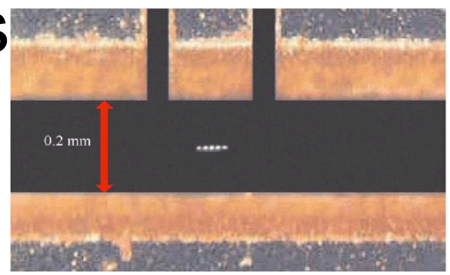
- NMR quantum computing
- Quantum computing with photons

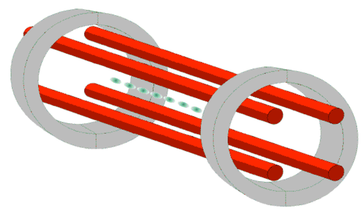




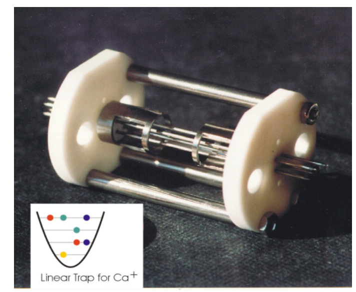


- Cavity QED
- Trapped ions





Blatt group, Innsbruck



Wineland group, Boulder

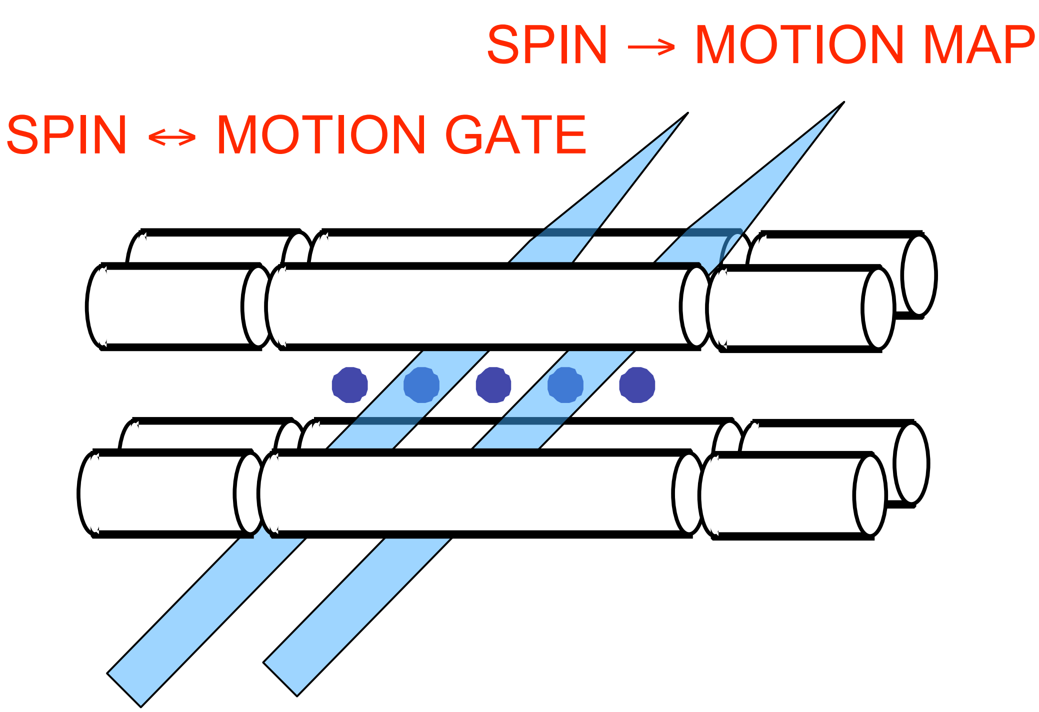
- Solid state (quantum dots, etc.)
- Neutral cold atoms and molecules

Trapped ion experimental groups pursuing Quantum Information Science:

- Aarhus
- Barcelona
- Berkeley
- Garching, MPQ
- Georgia Tech
- Griffiths University
- Innsbruck
- LANL
- London (Imperial)
- U. Maryland
- MIT
- NIST
- NPL, U.K.
- Osaka University
- Oxford
- Paris (Université Paris)
- PTB, Germany
- Sandia National Lab
- Siegen
- Simon Fraser University
- Singapore
- Sussex
- Ulm
- U. Washington
- Weizmann Institute

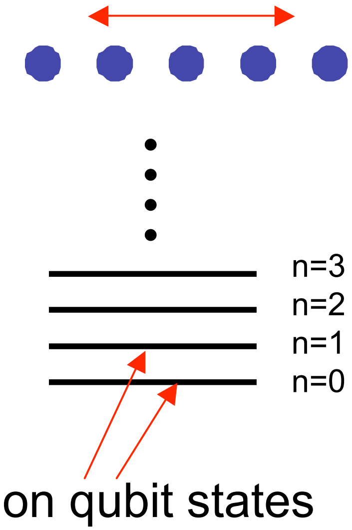
(■ U.S. groups)

Atomic Ion QI processing

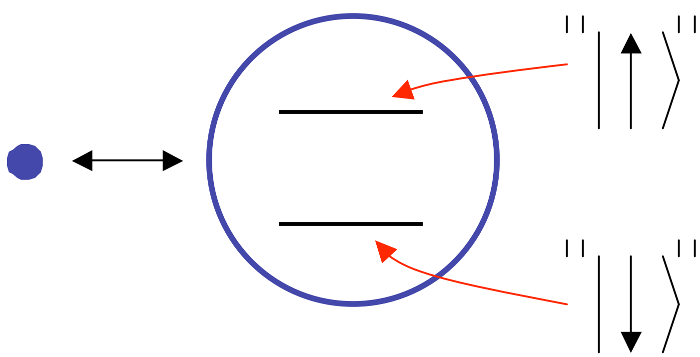


MOTON “DATA BUS”

(e.g., center-of-mass mode)



INTERNAL STATE QUBIT



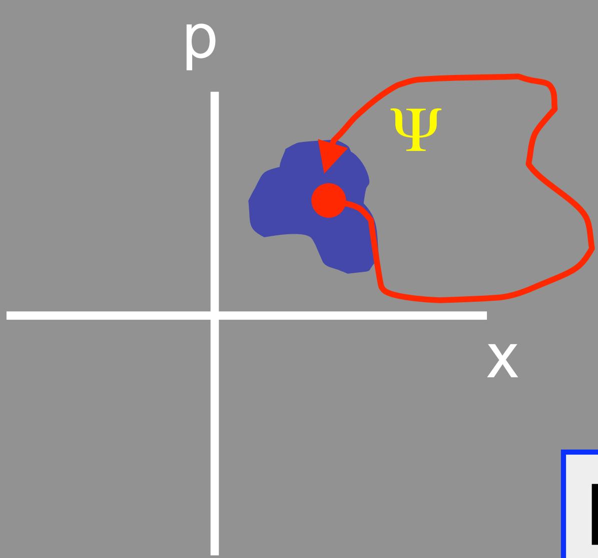
Optical qubits (e.g. Ca^+, λ = 729 nm)
- radiative decay $\tau \sim 1$ s

Hyperfine qubits
- $\tau \rightarrow \infty$ ($\tau_{coherence}$ > 10 min observed)

basic scheme: J.I. Cirac, P. Zoller, ’95

Multi-qubit phase gates

phase-space diagram for selected motional mode



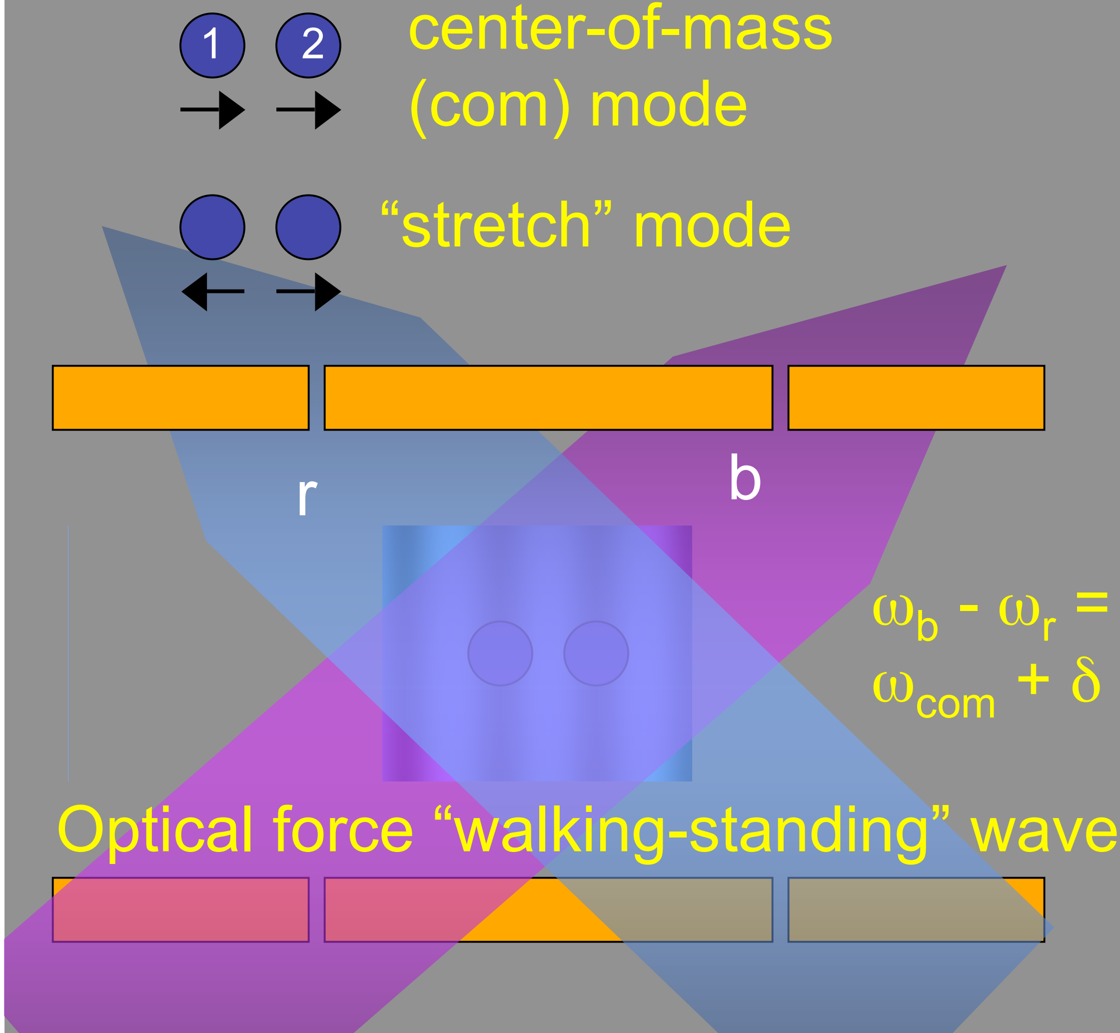
$\Psi \rightarrow e^{i\phi}\,\Psi$

$\phi \propto$ enclosed area

Phase gate:

- make force state-dependent
- use optical dipole forces

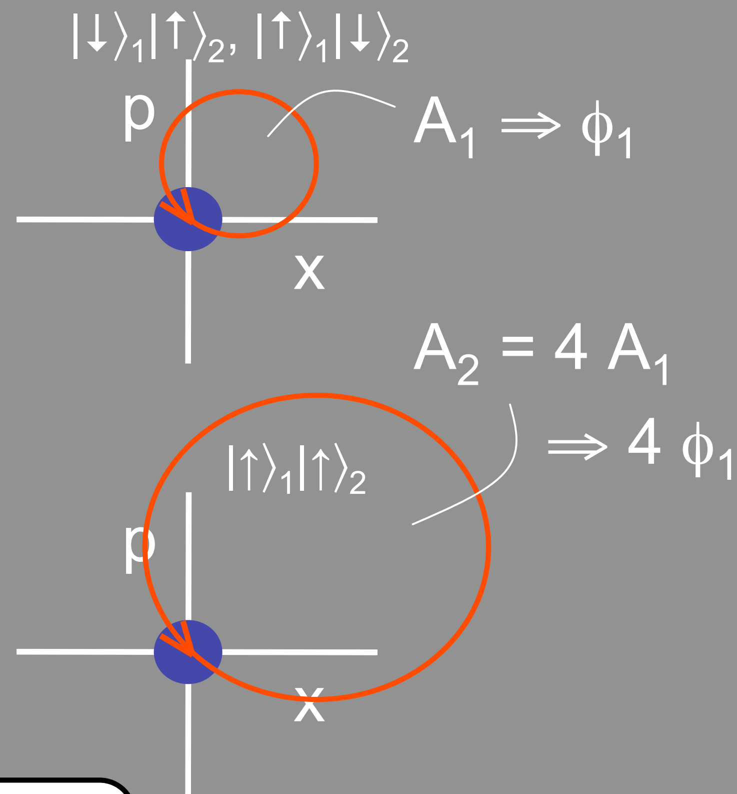
Example: z-basis phase gates



assume dipole force

$F_{1\uparrow} = F_{2\uparrow} = F_{\uparrow}$

$F_{1\downarrow} = F_{2\downarrow} = 0$



$$\Psi = \alpha|\downarrow\rangle|\downarrow\rangle + \beta|\downarrow\rangle|\uparrow\rangle + \gamma|\uparrow\rangle|\downarrow\rangle + \delta|\uparrow\rangle|\uparrow\rangle \rightarrow$$

$$\alpha|\downarrow\rangle|\downarrow\rangle + e^{i\phi_1}\beta|\downarrow\rangle|\uparrow\rangle + e^{i\phi_1}\gamma|\uparrow\rangle|\downarrow\rangle + e^{4i\phi_1}\delta|\uparrow\rangle|\uparrow\rangle$$

z rotations + phase gate

example: z-basis phase gates

<u>General formalism</u>:

Milburn, Schneider, James (1999)

Sørensen & Mølmer (1999, 2000)

Solano, de Matos Filho, Zagury (1999)

<u>Experiments</u>:

Sackett *et al*. (2000)

x-y basis states:

Leibfried *et al*. (2003) (F = 0.97)

z basis states:

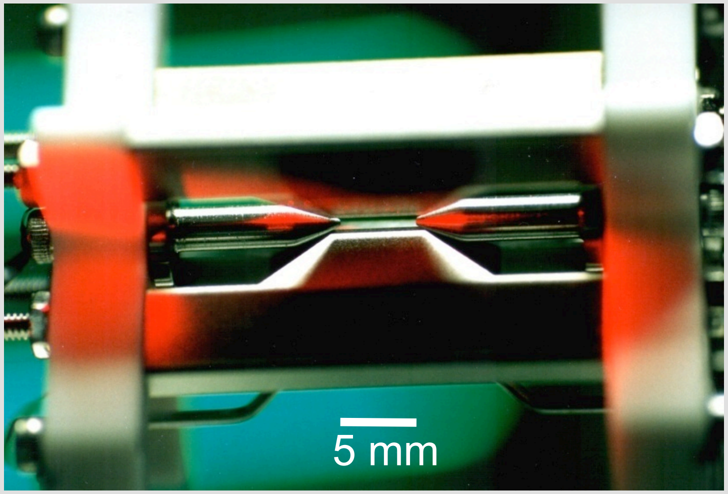
Monroe group (2005) x-y basis

Oxford (2005) z basis

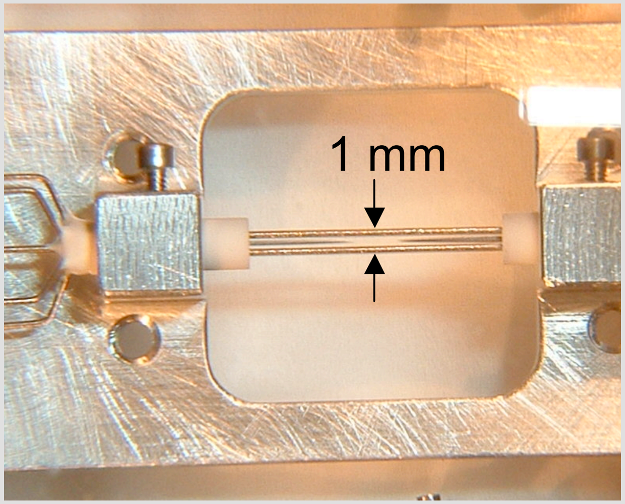
Innsbruck (2008) x-y basis (F = 99.3)

Traps

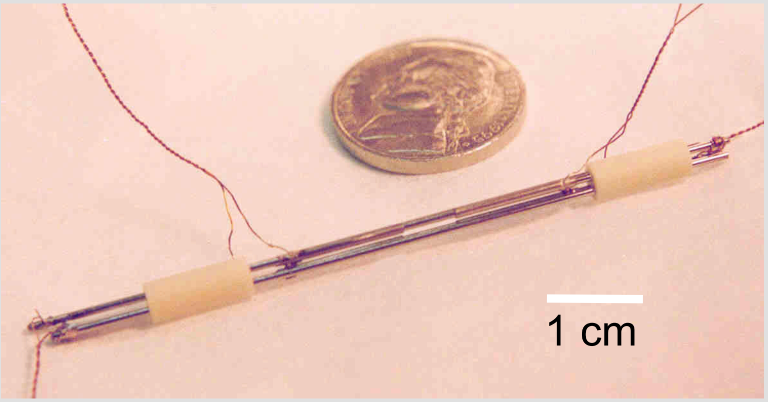
motion frequencies, gate speeds ∝ V_{RF}/md^2 ⇒ make d small



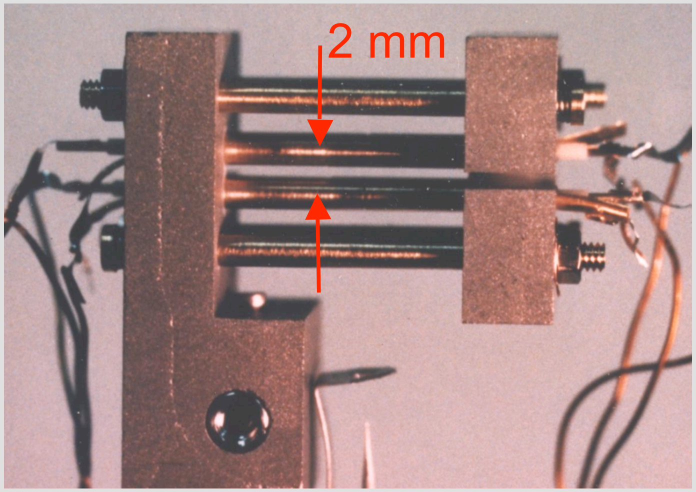
Innsbruck



U. Md.

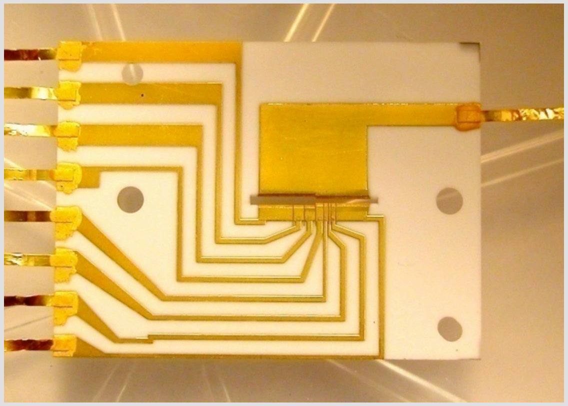


LANL

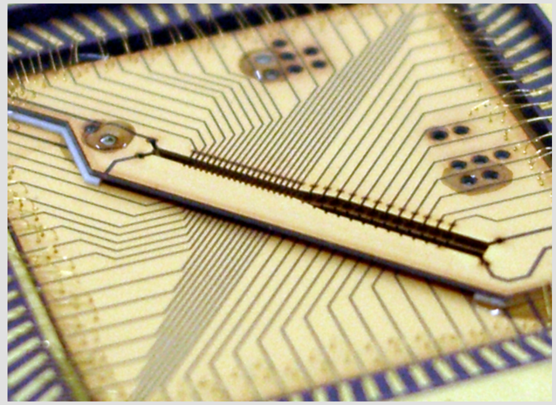


NIST

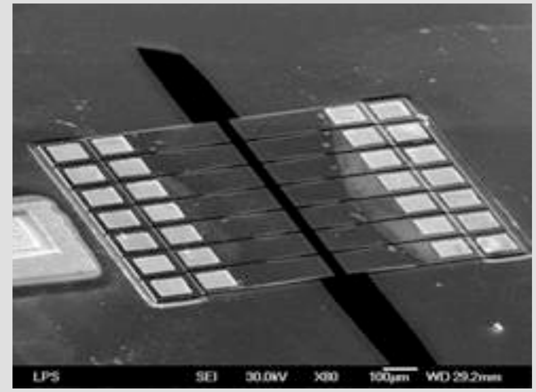
Representative multizone traps



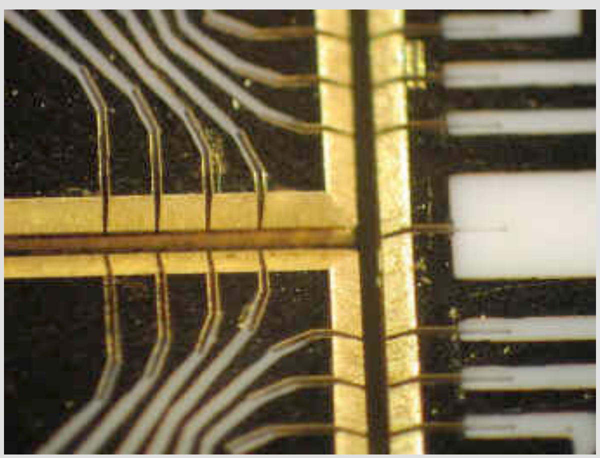
NIST, Au on Al_2O_3
2-layer, 6 zones



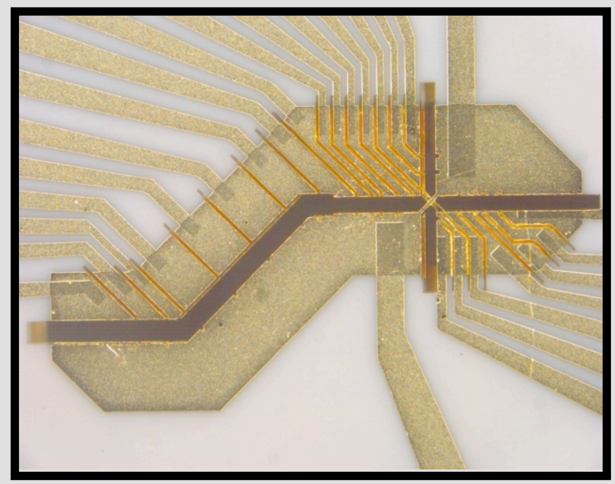
Ulm (Schmidt-Kaler) Au
on $Al_2O_{3,}$ 2-layer, 31 zones



Monroe group, GaAs
on AlGaAs, 2-layer, 5 zones

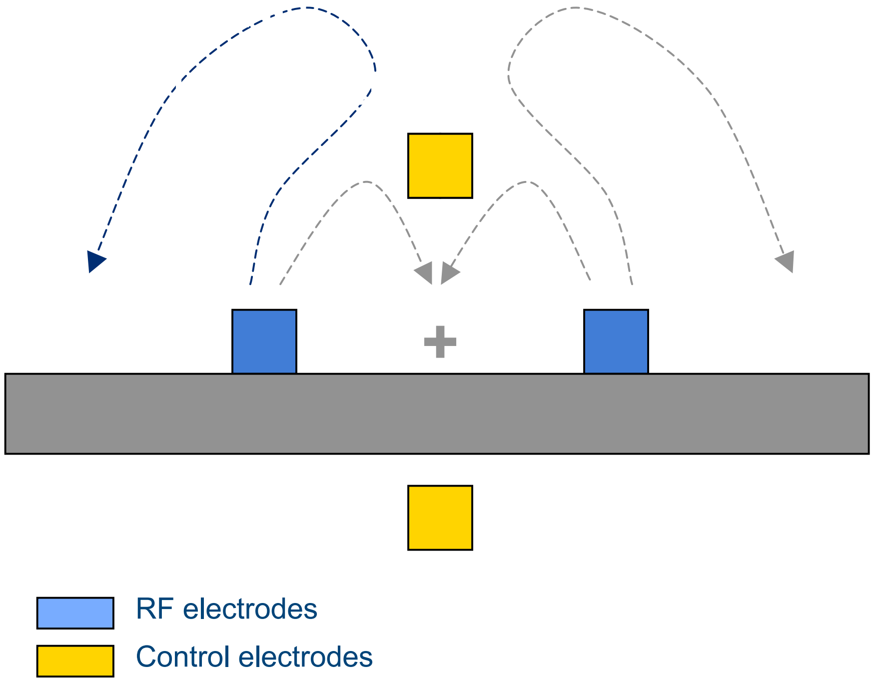


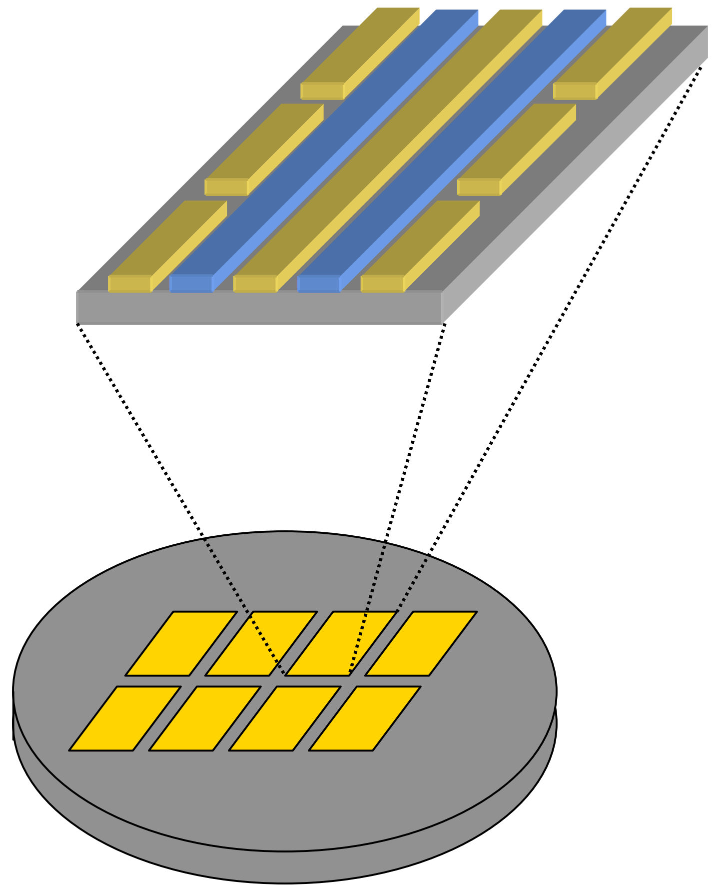
Monroe group, Au on Al_2O_3
3-layer, 9 zone “T” trap



NIST, Au on Al_2O_3
2-layer,18 zones

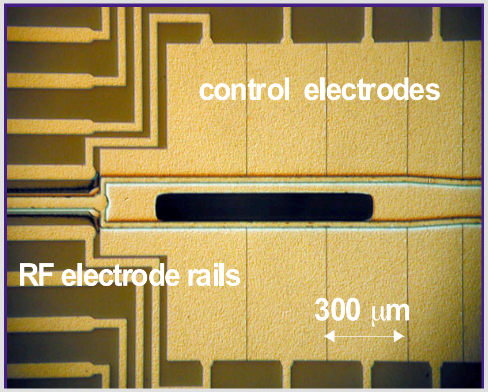
Further scaling ?



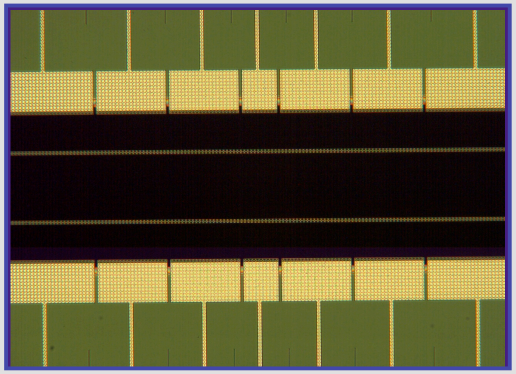


J. Chiaverini *et al.*, Quant. Inform. Comp. 5, 419 (2005)

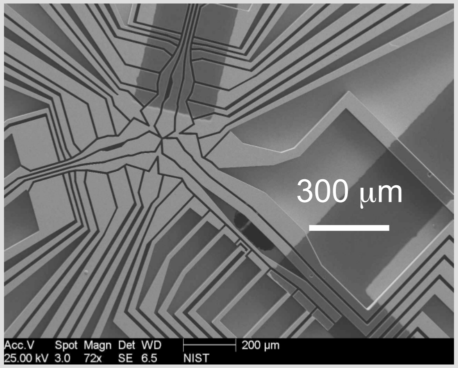
Surface-electrode trap examples



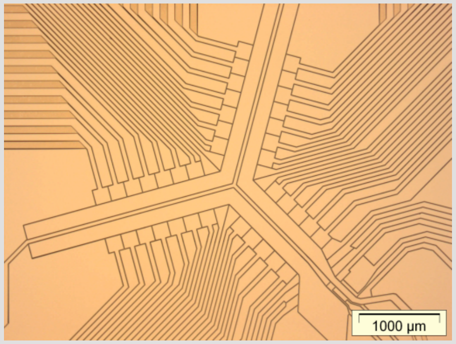
Lucent
Al on Si
17 zones



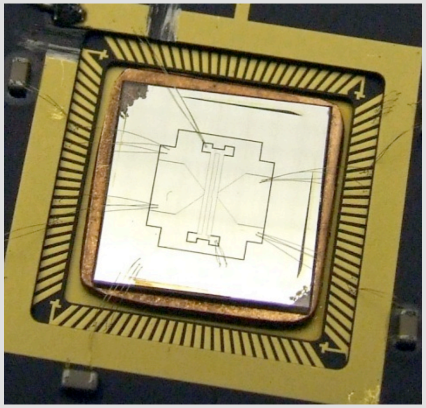
Sandia
W on Si
5 zones



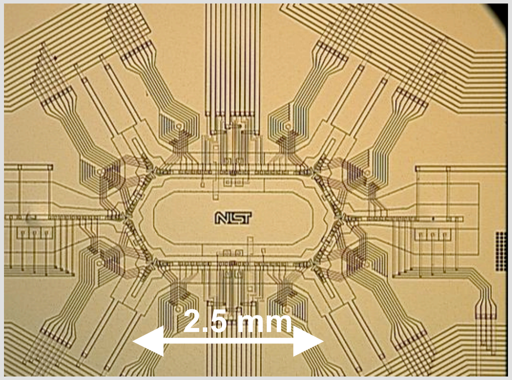
NIST
B-doped Si
23 zones



Ulm,
Au on Al_2O_3
24 zones



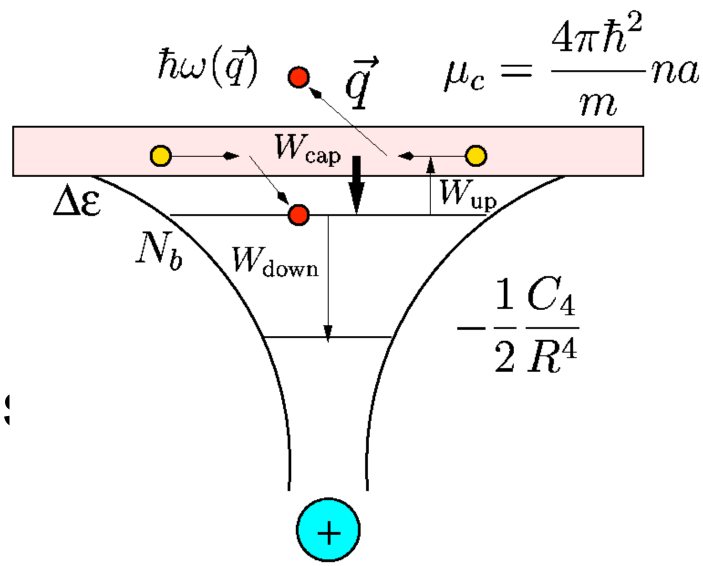
MIT
Ag on
Sapphire
1 zone



NIST
Au on quartz
~200 zone
“racetrack”

Atom-ion scattering & doped ultracold samples

- Why ultracold atom-ion scattering ?
  - Possible to cool ions by elastic collisions with ultracold atoms
  - Study of charge transfer in the ultracold regime
    - Hopping conductivity at ultralow temperature
- Several experimental efforts are under way
  - $NaCa^+$ at UConn (W. Smith)
  - $RbBa^+$ in Innsbruck (J. Denschlag)
  - Yb + Yb^+ at MIT (V. Vuletic)
- Formation of Mesoscopic "ion cluster"
  - Ions in BEC captures several atoms



Atom-ion Scattering

- Consider “identical parents” (e.g. Na)
  - Two possible outcomes

$$\mathrm{Na{+}Na^+} \rightarrow \begin{cases} \mathrm{Na}_2^+(^2\Sigma_g^+) \quad \text{or} \quad \mathrm{Na}_2^+(^2\Sigma_u^+) & \rightarrow \mathrm{Na} + \mathrm{Na}^+ \,, \quad \text{elastic collision} \,, \\ \mathrm{Na}_2^+(^2\Sigma_g^+) \quad \text{and} \quad \mathrm{Na}_2^+(^2\Sigma_u^+) & \rightarrow \mathrm{Na}^+ + \mathrm{Na} \,, \quad \text{charge transfer} \,. \end{cases}$$

- Long-range polarization potential

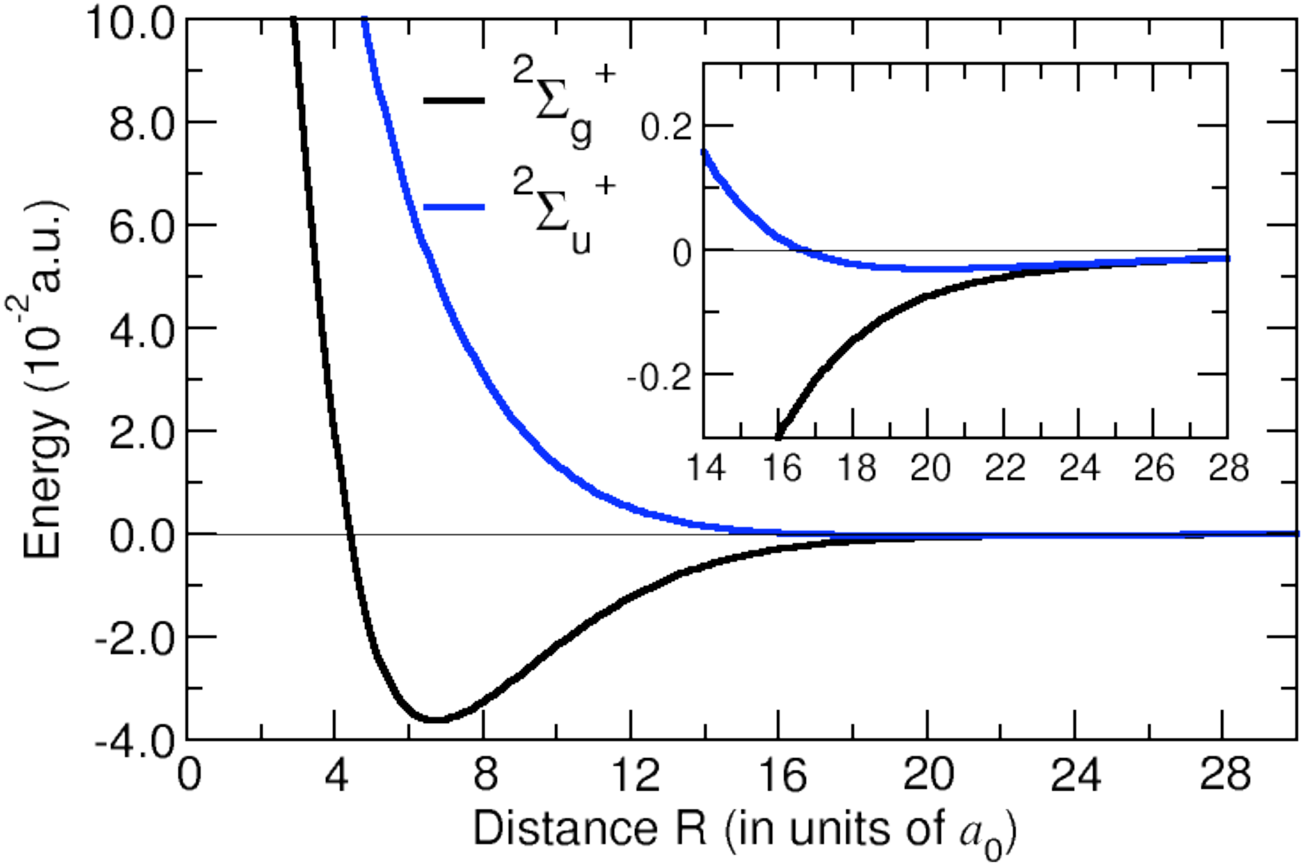
$$V_{g,u}(R) \sim -\frac{1}{2}\left[\frac{C_4}{R^4} + \frac{C_6}{R^6} + \frac{C_8}{R^8}\right] \mp \frac{1}{2} A R^{\alpha} e^{-\beta R}\left[1 + \frac{B}{R} + \mathcal{O}\left(\frac{1}{R^2}\right)\right]$$

  - Many partial waves even at ultralow T

Potentials and Phase shifts

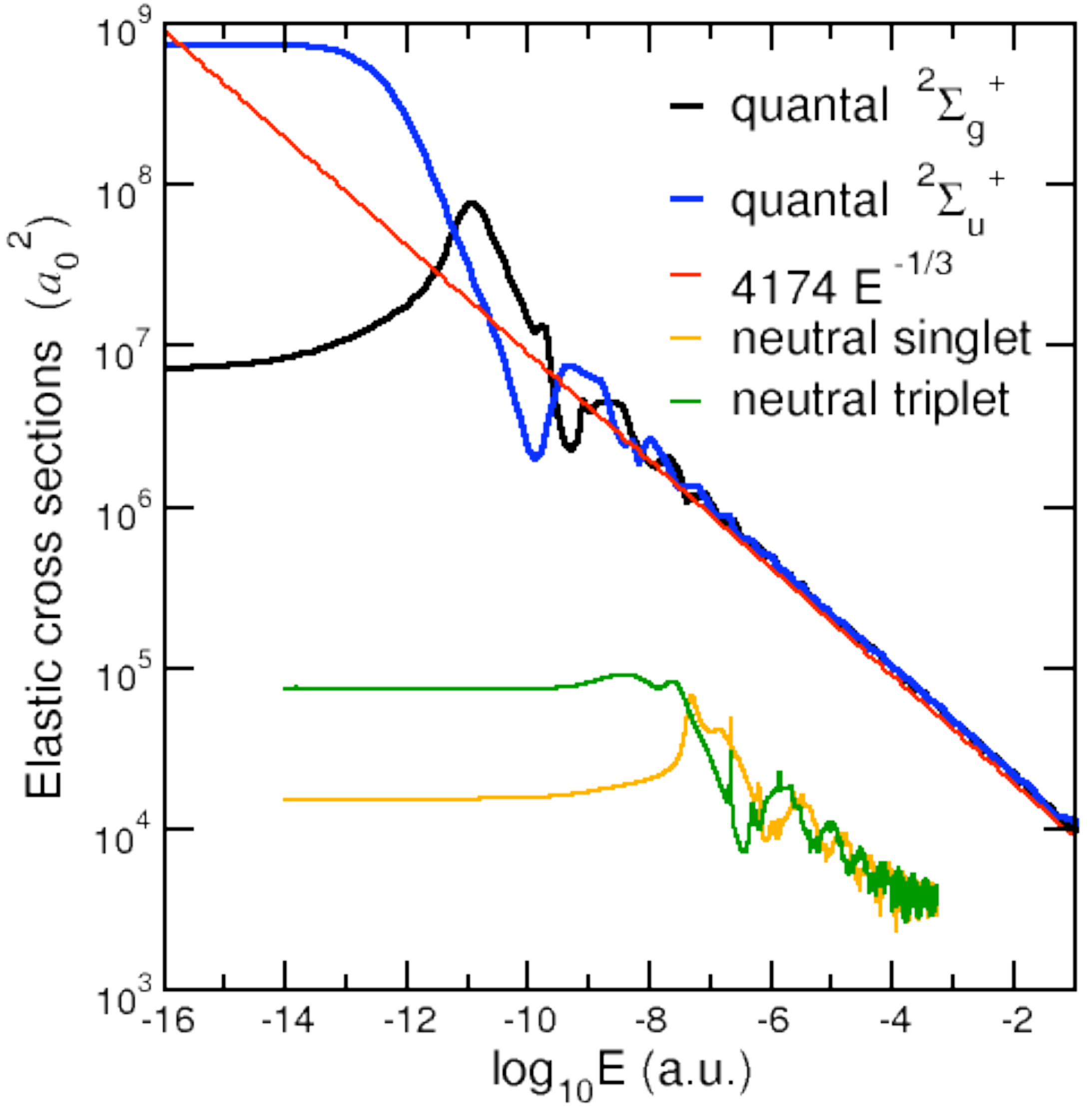
- Many partial waves even at ultralow *T*

$$\eta_l \simeq -\frac{\mu}{\hbar^2}\int_{R_0}^{\infty} dR \frac{V(R)}{\sqrt{k^2-(l+\frac{1}{2})^2/R^2}} \simeq \frac{\pi\mu^2 C_4}{4\hbar^4}\frac{E}{l^3}$$



Results

- Approximation: quite good



Charge Transfer

- Cross section

$$\sigma_{\text{ch.}} = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) \sin^2(\eta_l^g - \eta_l^u)$$

- Ungerade much shallower than gerade
  - Almost like a single curve
  - Langevin scattering cross section

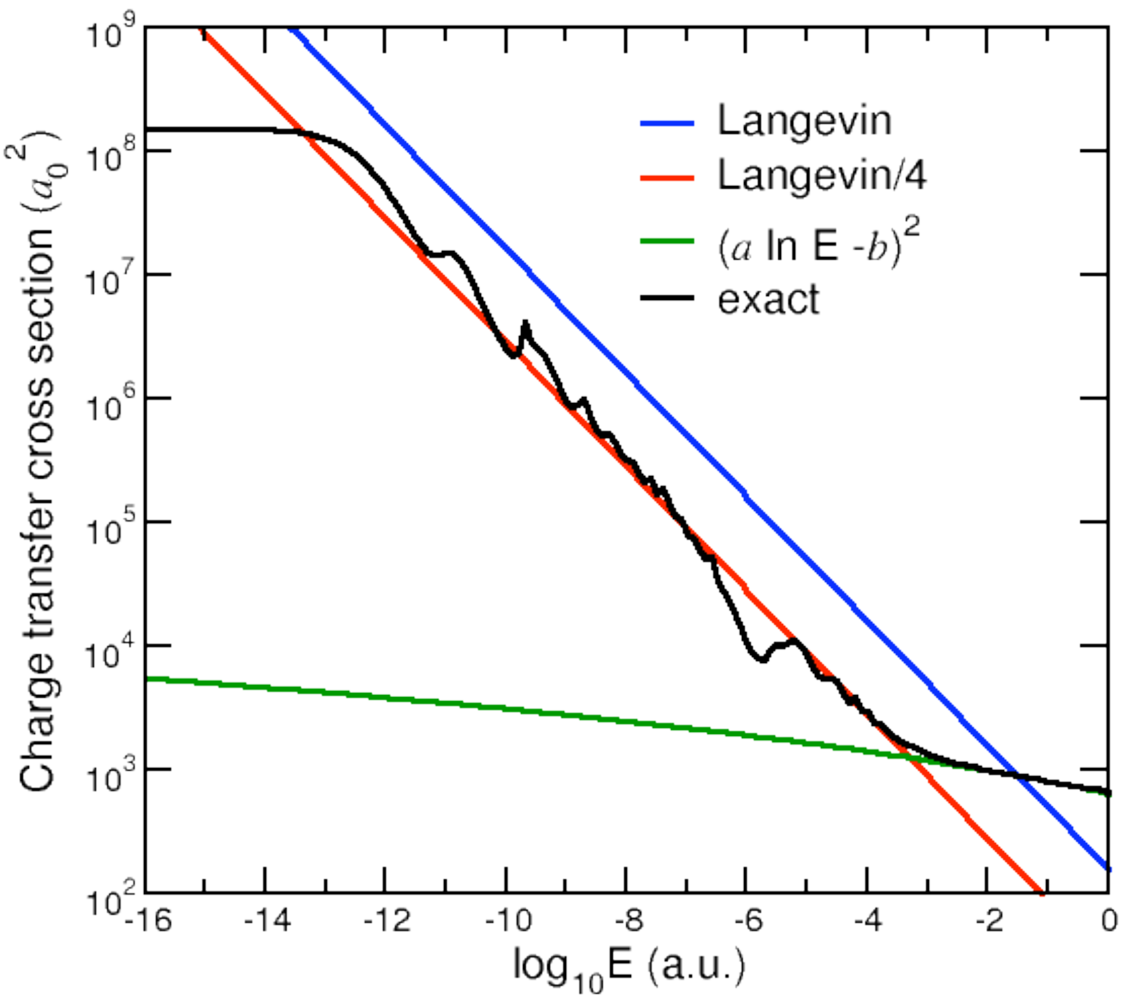
$$\sigma_{\text{Langevin}} \sim \pi \sqrt{2C_4} E^{-1/2}$$

- At large energy, we expect

$$\sigma_{\text{ch}} \sim (a \ln E - b)^2 \text{a.u.}$$

Charge Transfer: results

- Langevin good over large energy range

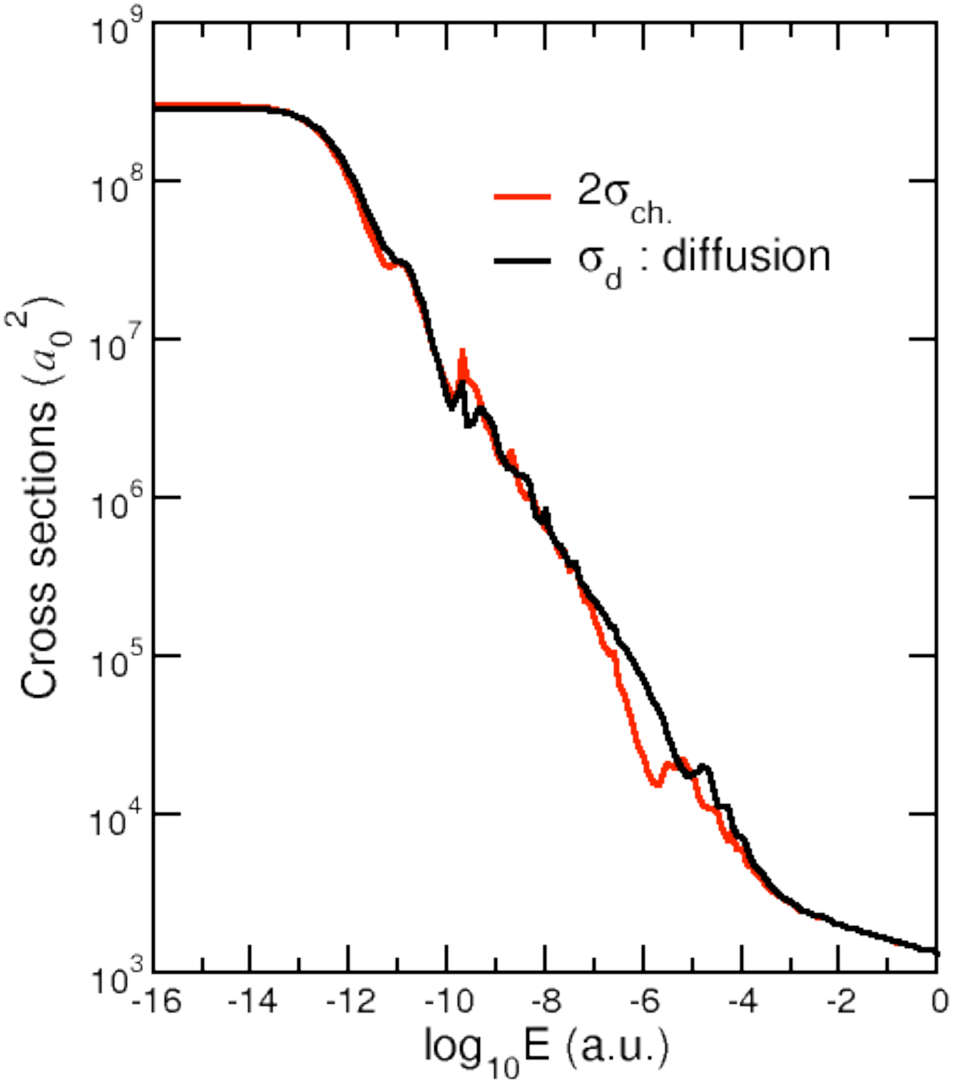


Approximation

- Many partial waves

$$\eta_l^{g,u} \simeq \eta_{l+1}^{g,u}$$

$$\sigma_d \simeq 2\sigma_{\rm ch}$$



Mobility

- Ion mobility

$$\mu_{\rm ion} = \frac{eD_{\rm ion}}{k_B T}$$

- Diffusion coefficient

$$D_{\rm ion} = \frac{3\sqrt{\pi}}{16(n_{\rm ion}+n_{\rm at})}\sqrt{\frac{2k_BT}{\mu}}\frac{1}{\langle\sigma_d\rangle} \simeq \frac{3\sqrt{\pi}}{16n_{\rm at}}\sqrt{\frac{2k_BT}{\mu}}\frac{1}{\langle\sigma_d\rangle}$$

$$\langle\sigma_d\rangle = \frac{1}{2}\int_0^\infty dx\, x^2 \exp(-x)\sigma_d(x) \qquad x = E/k_BT$$

$$\mu_{\rm ion} \simeq \frac{35.9\,\zeta}{\sqrt{MC_4}}\,{\rm cm}^2{\rm V}^{-1}{\rm s}^{-1} \simeq 0.59\,\zeta\,{\rm cm}^2{\rm V}^{-1}{\rm s}^{-1}$$

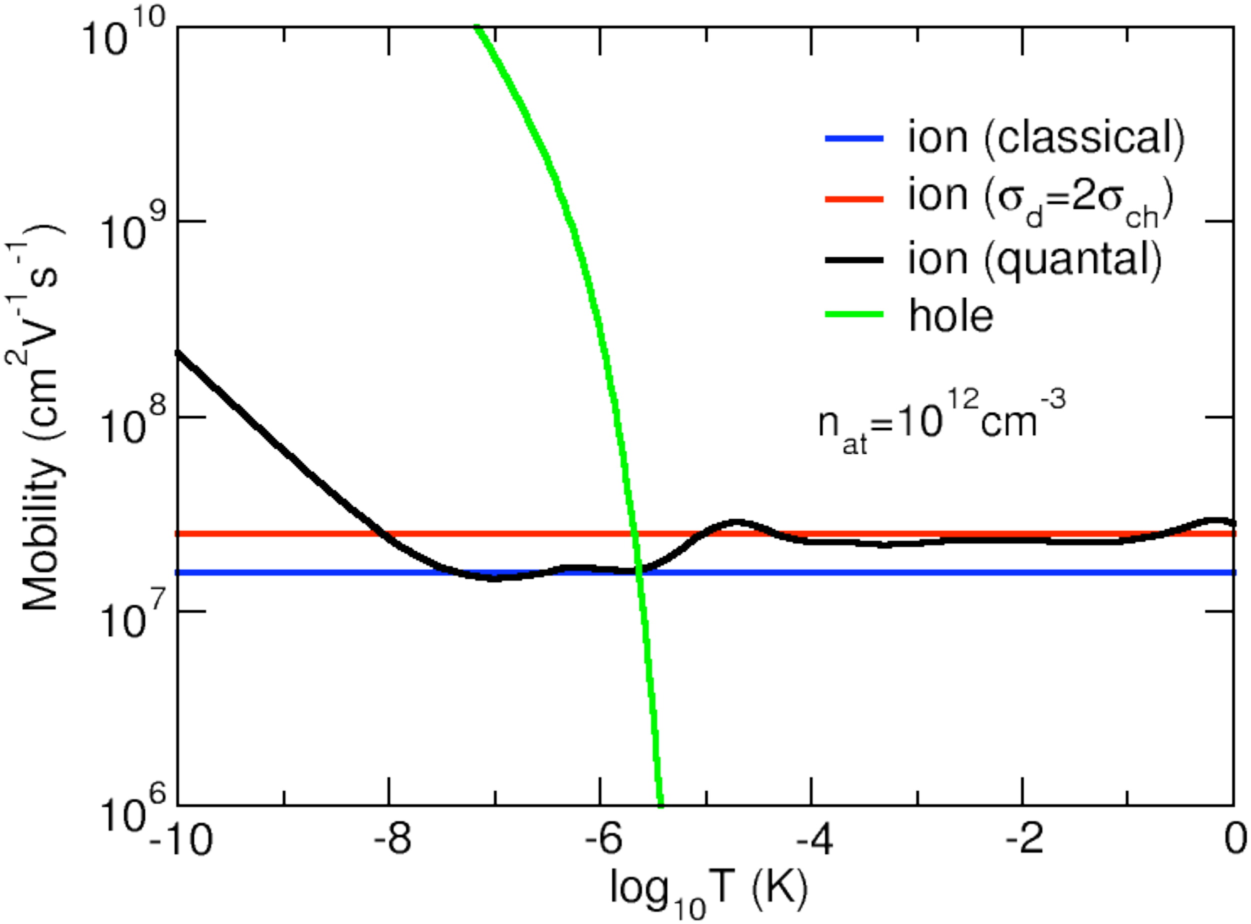
$$\zeta = n_{\rm std}/n_{\rm at} \qquad n_{\rm std} = 2.69\times10^{19}{\rm cm}^{-3}$$

Results

- Approximation for charge transfer

$$\mu_{\rm ion} = \frac{e\ \tau}{m_{\rm ion}} \simeq 0.92\ \zeta\ {\rm cm^2V^{-1}s^{-1}}$$

$$\tau = 1/n_{\rm at}\sigma_d v$$



Conductivity

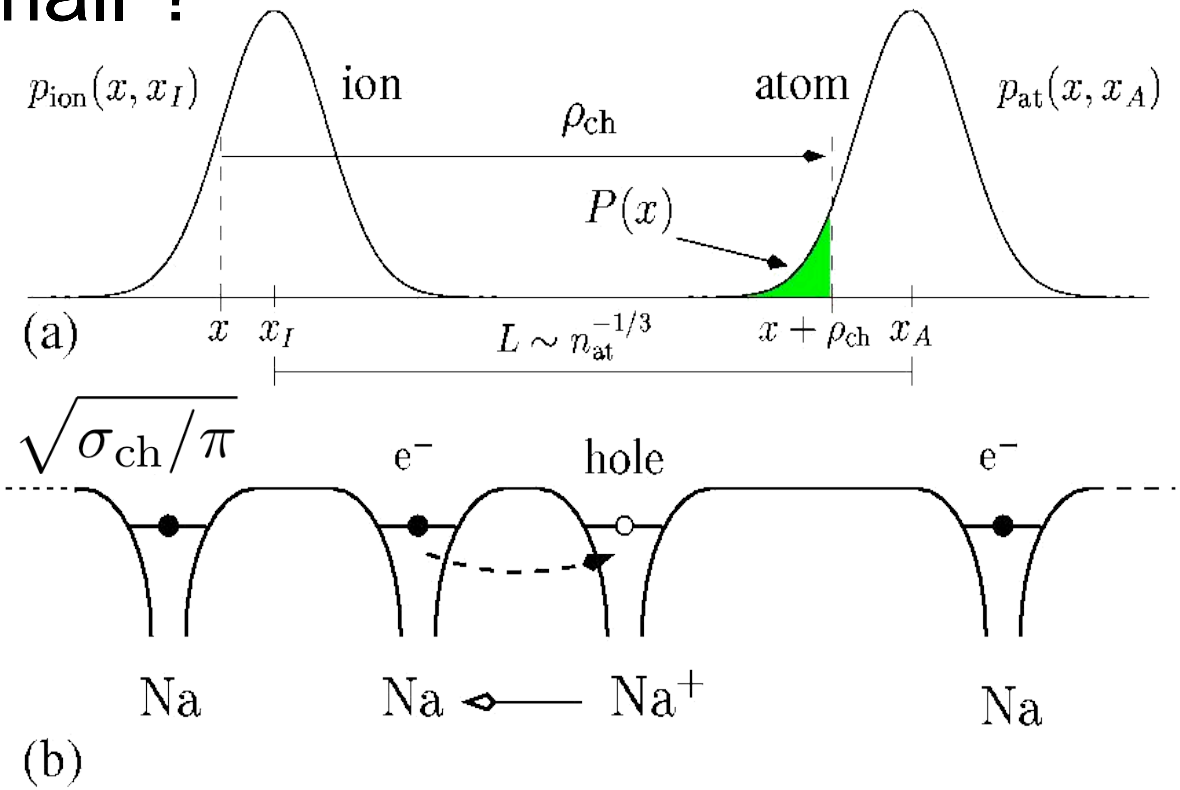
- If T becomes small ?

$$\lambda_T = (2\pi\hbar^2/mk_BT)^{1/2}$$

$$\lambda_T = 6.88\ T^{-1/2}a_0$$

$$\sigma_{\rm ch} \equiv \pi\rho_{\rm ch}^2 \quad \text{or} \quad \rho_{\rm ch} = \sqrt{\sigma_{\rm ch}/\pi}$$

$$\rho_{\rm ch} = 50.34\ T^{-1/4}a_0$$



$$p_{\rm at}(x, x_A) = \frac{1}{\sqrt{2\pi}\lambda_T}\exp\left(-\frac{(x-x_A)^2}{2\lambda_T^2}\right),$$

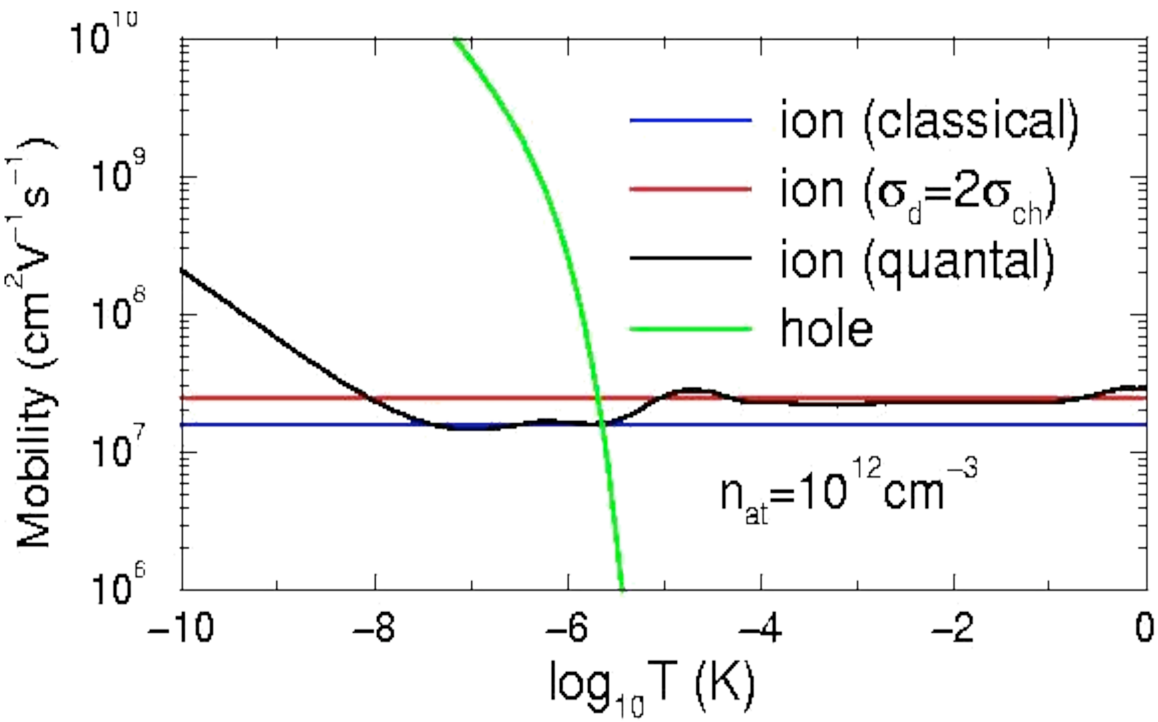
$$p_{\rm ion}(x, x_I) = \frac{1}{\sqrt{2\pi}\lambda_T}\exp\left(-\frac{(x-x_I)^2}{2\lambda_T^2}\right)$$

Conductivity

- What happens when T becomes small ?
  - Current is $$j = \sigma_{\rm cond}\mathcal{E}$$

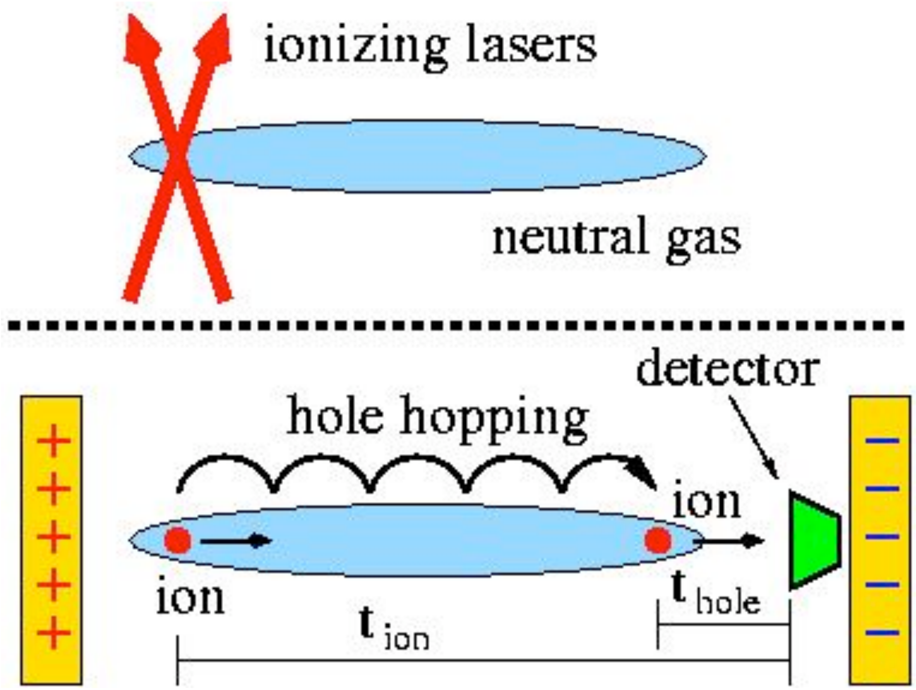
$$\sigma_{\rm cond} = n_h\; e\; \mu_h + n_{\rm ion}\; e\; \mu_{\rm ion} = n_{\rm ion}\; e\; \mu_{\rm tot}$$

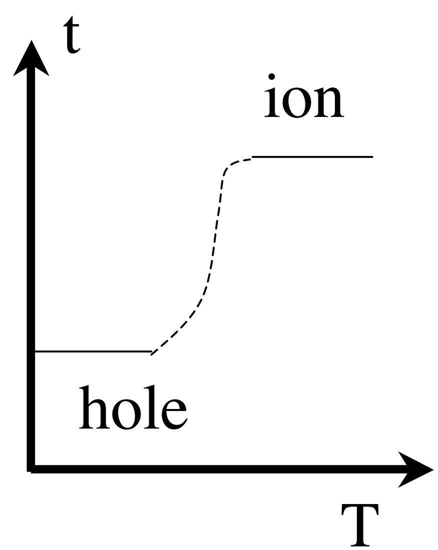
$$\mu_{\rm tot} = \mu_h + \mu_{\rm ion}$$



Possible experiment

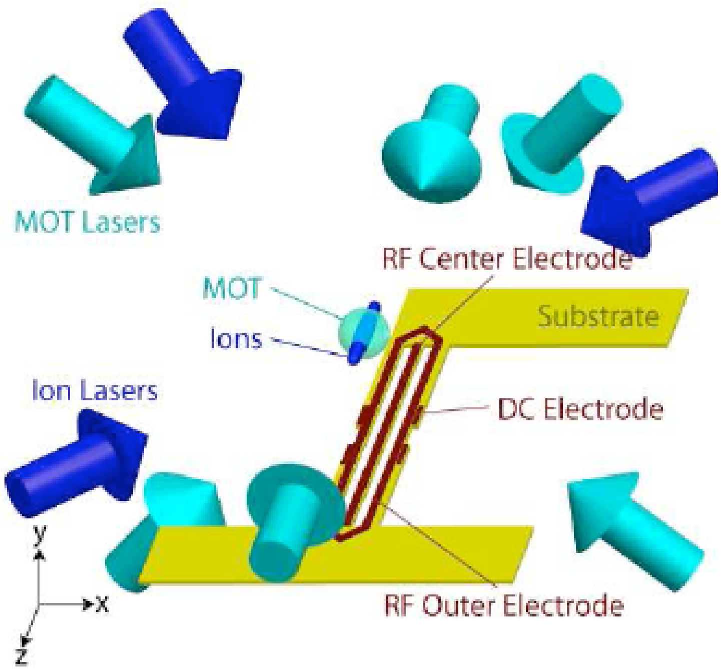
- A “thought” experiment

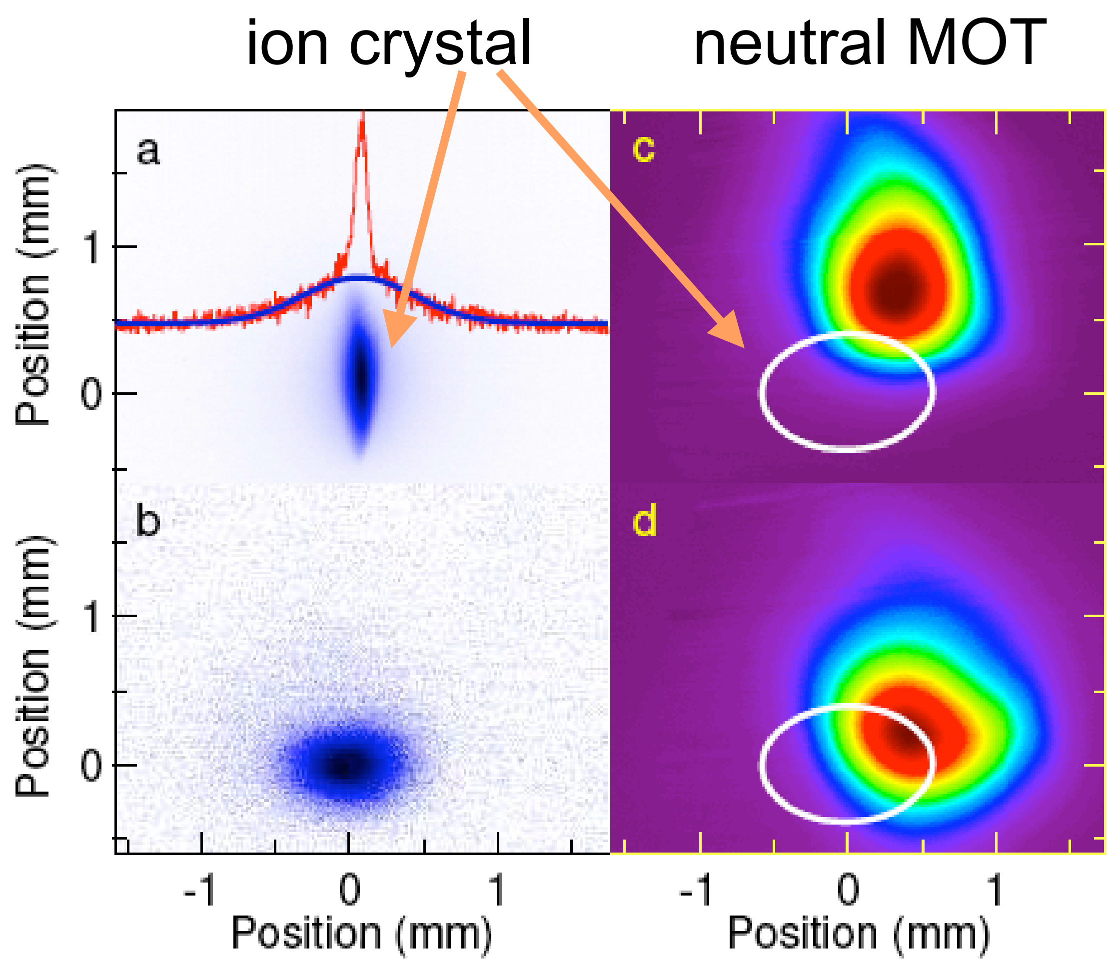




Yb + Yb$^+$ at MIT

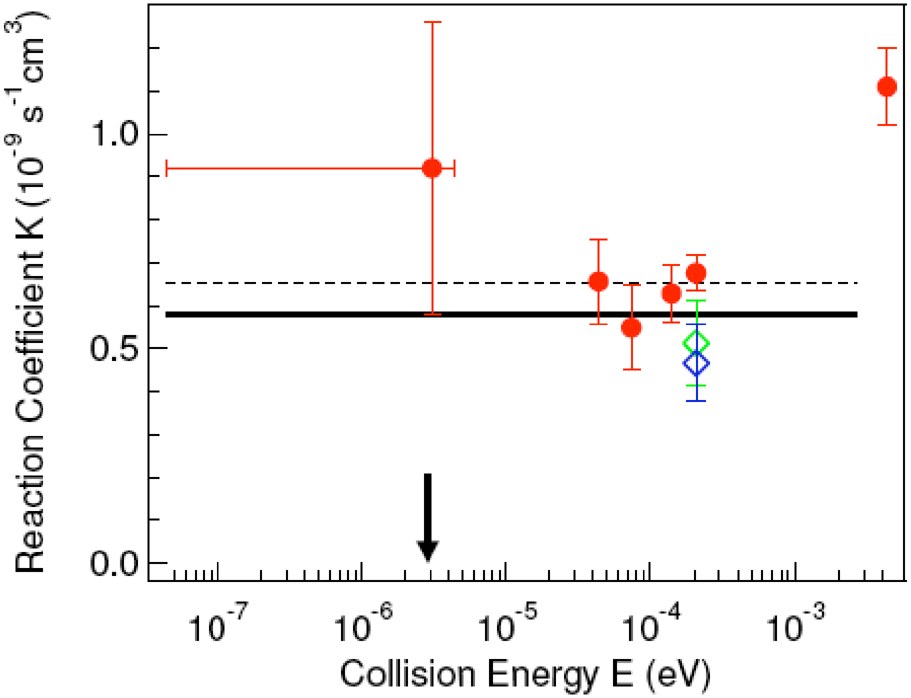
- Dual trap: $^{172}Yb^+$ and ^{174}Yb (and other)

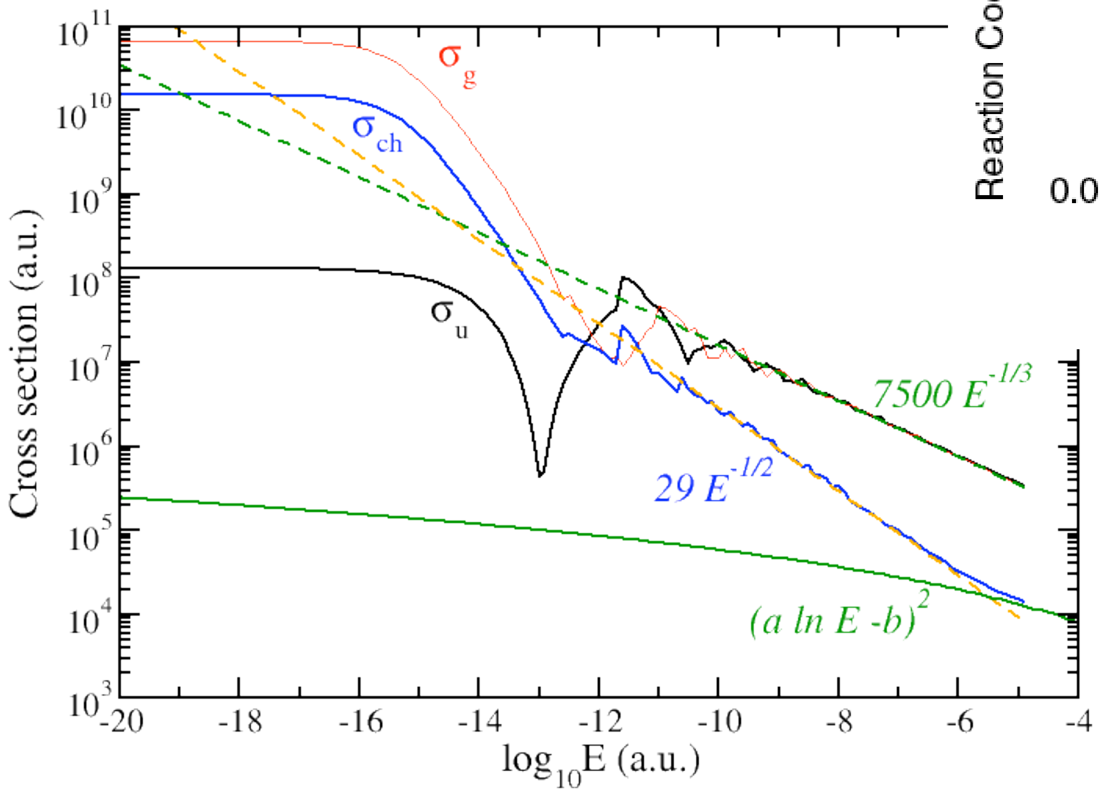




Results

- In agreement with Langevin σ

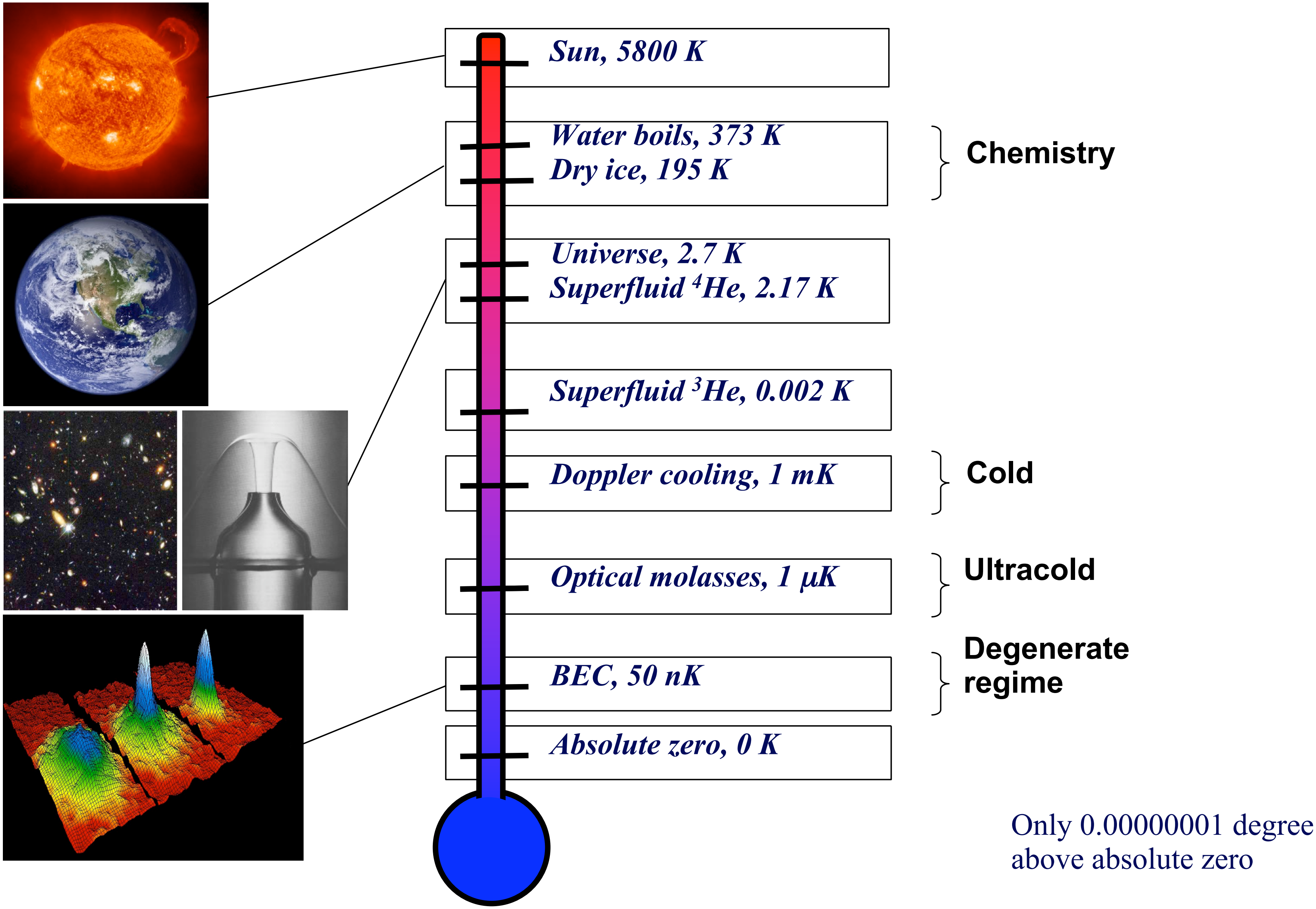




Back to atoms

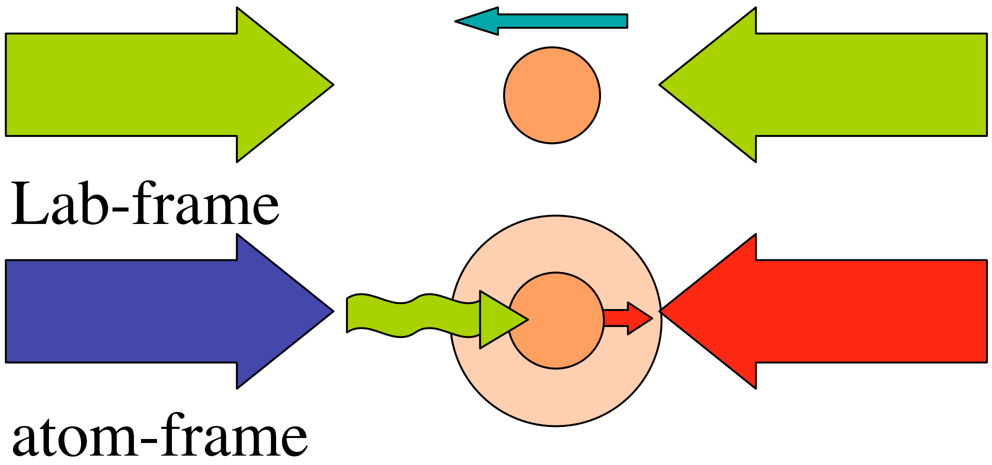
- What is ultracold ?
- How do we get there ?
  - Cooling + trapping
- Review of scattering
  - Scattering length
  - Feshbach resonances
- Real systems

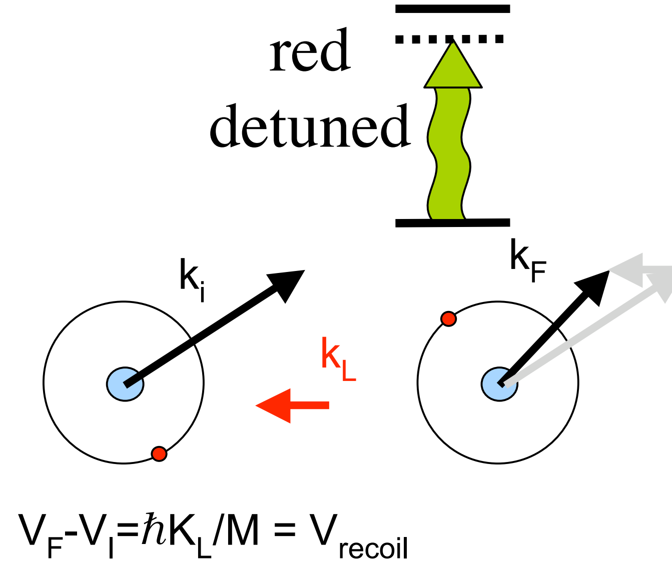
How cold is COLD?



Cooling mechanisms

- Laser cooling





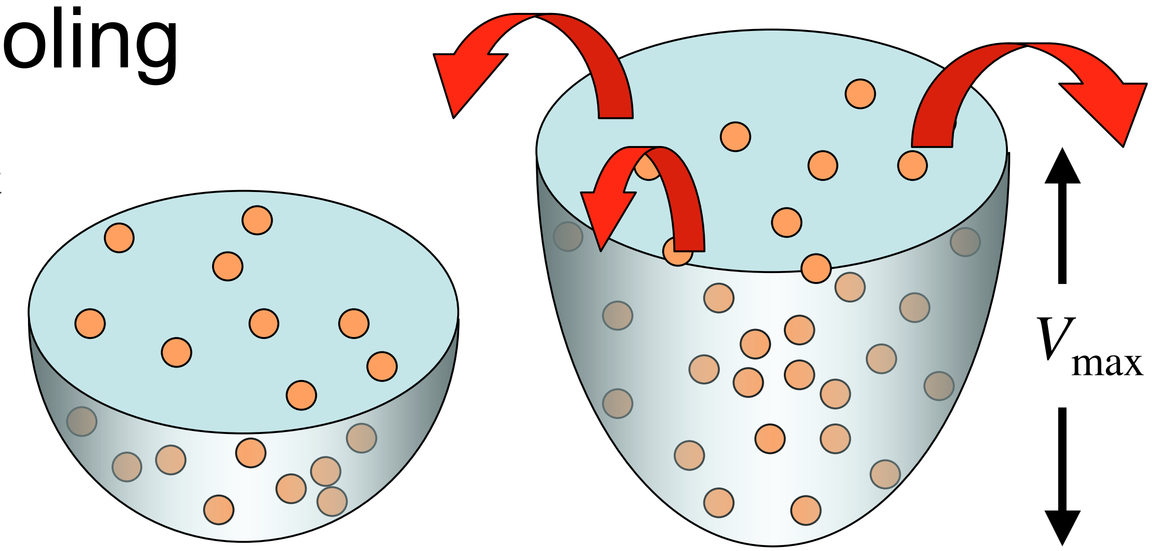
- Evaporative cooling

$$\overline{E} \sim \frac{3}{2} k_B T < V_{\max}$$

  – smaller V_{max}

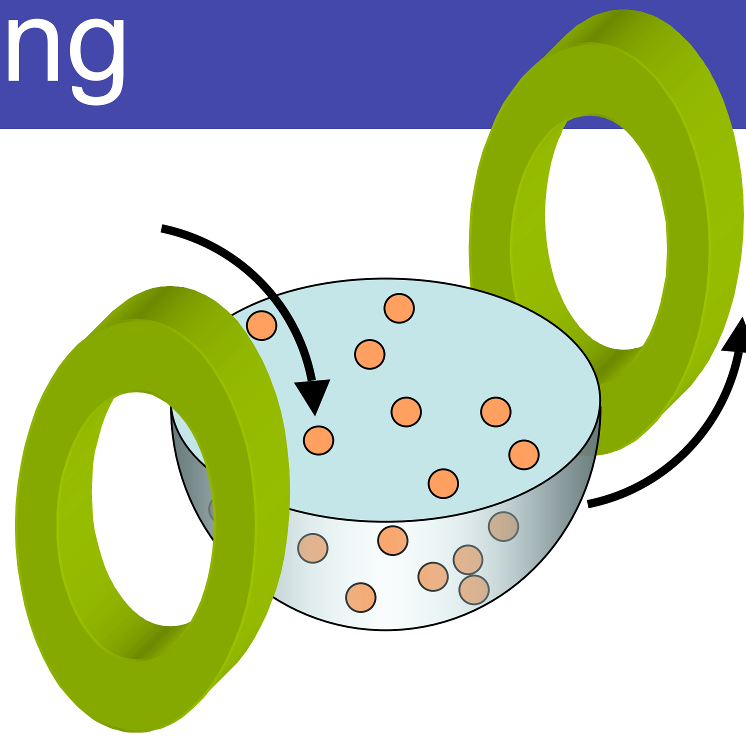
  $\Rightarrow$ lower T

  – atoms are lost



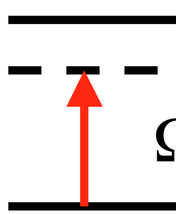
Trapping

- Two main approaches
- Magnetic traps

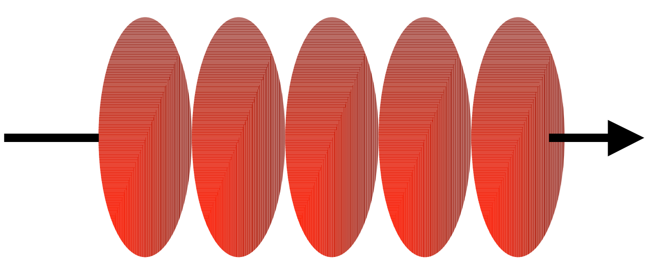


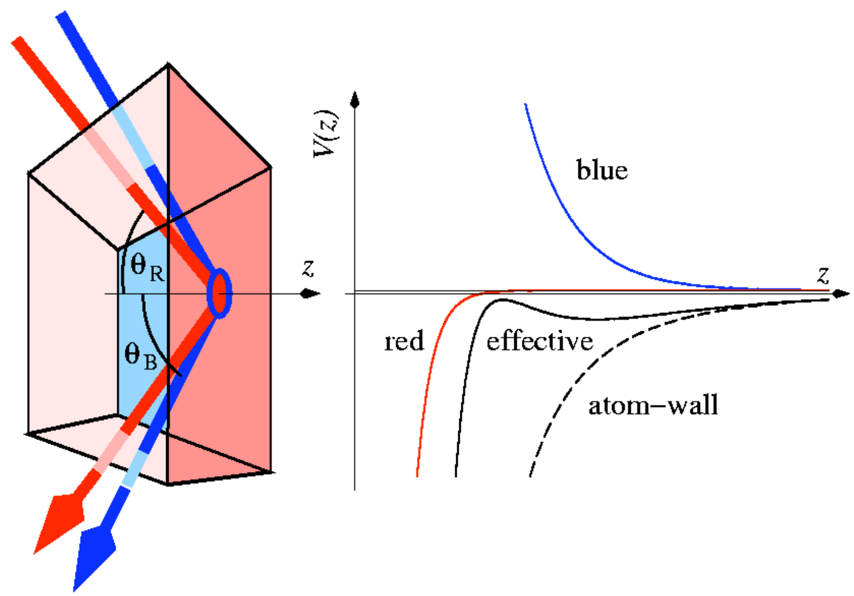
$$\mathbf{F} = -\mu \cdot \mathbf{B} = -\nabla V$$

- Optical traps
  - Dipole force



$$V_{\text{dip}} \simeq \frac{\hbar\Omega^2}{4\delta} = \frac{Id^2}{8\hbar\epsilon_0\delta}$$





Scattering: brief review

- Relative motion: $$\left[\nabla_{\mathbf{r}}^2 + k^2 - U(\mathbf{r})\right]\psi(\mathbf{r}) = 0$$

$$\mathbf{p} = \hbar\mathbf{k} \qquad E = \frac{p^2}{2\mu} = \frac{\hbar^2 k^2}{2\mu} \qquad U(\mathbf{r}) = \frac{2\mu}{\hbar^2}V(\mathbf{r})$$

- Solution for $\lim_{r\to\infty} r^2 V(\mathbf{r}) = 0$

$$\psi_{\mathbf{k}}^{(+)}(\mathbf{r}) \rightarrow A\left\{\exp(i\mathbf{k}\cdot\mathbf{r}) + f(k,\theta,\phi)\frac{\exp(ikr)}{r}\right\}$$

- Scattering amplitude and cross section

$$\frac{d\sigma}{d\Omega} = |f(k,\Omega)|^2 \qquad \sigma_{tot} = \int d\Omega \frac{d\sigma}{d\Omega}$$

Partial waves

- Central potential $V(\mathbf{r})=V(r)$

$$H = -\frac{\hbar^2}{2\mu}\nabla_{\mathbf{r}}^2 + V(r) = -\frac{\hbar^2}{2\mu}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{\mathbf{L}^2}{\hbar^2 r^2}\right] + V(r)$$

$$\psi_{\mathbf{k}}^{(+)}(k,\mathbf{r}) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{+\ell} c_{\ell m}(k)\frac{u_\ell(k,r)}{r}Y_{\ell m}(\theta,\phi)$$

$$\mathbf{L}^2 Y_{\ell m}(\theta,\phi) = \ell(\ell+1)\hbar^2 Y_{\ell m}(\theta,\phi)$$

$$\left[\frac{d^2}{dr^2} + k^2 - \frac{\ell(\ell+1)}{r^2} - U(r)\right]u_\ell(k,r) = 0$$

$$u_\ell(k,r) \to A_\ell(k)\sin\left(kr - \frac{1}{2}\ell\pi + \delta_\ell(k)\right)$$

Phase shift

- It gives information about the interaction

$$\psi_{\mathbf{k}}^{(+)}(k,\mathbf{r}) = A(k)\sum_{\ell=0}^{\infty}\frac{(2\ell+1)}{kA_\ell(k)}i^\ell \exp(i\delta_\ell)\frac{u_\ell(k,r)}{r}P_\ell(\cos\theta)$$

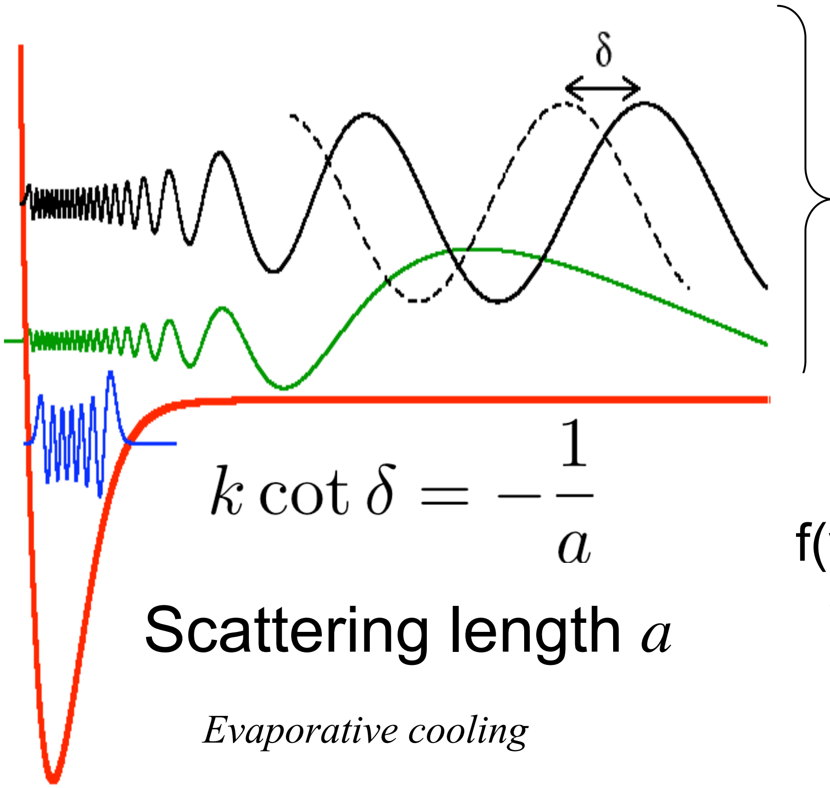
$$f(k,\theta) = \frac{1}{k}\sum_{\ell=0}^{\infty}(2\ell+1)e^{i\delta_\ell(k)}\sin\delta_\ell(k)P_\ell(\cos\theta)$$

$$\sigma_{\text{tot}}(k) = \frac{4\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\sin^2\delta_\ell(k)$$

- Low energy: s-wave (l = 0)

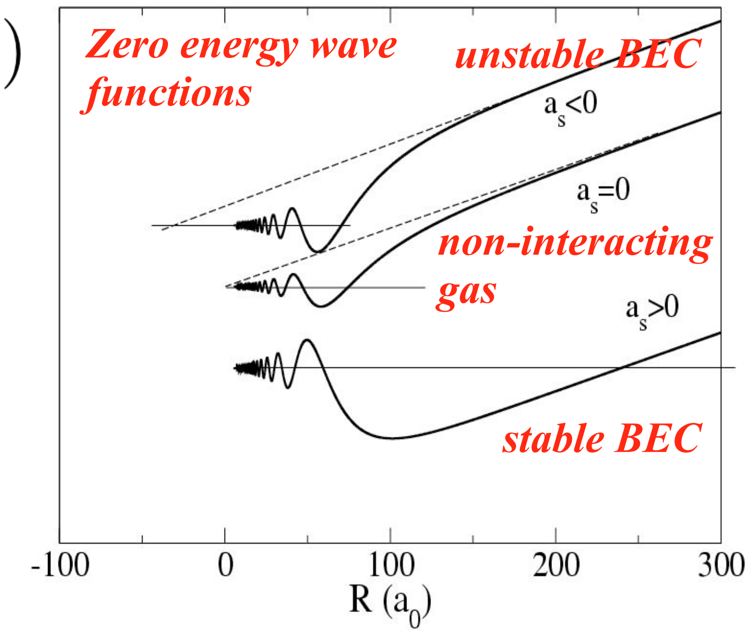
Collisions between ultracold atoms

- Wave function $u(R) \propto \sin(kR + \delta)$



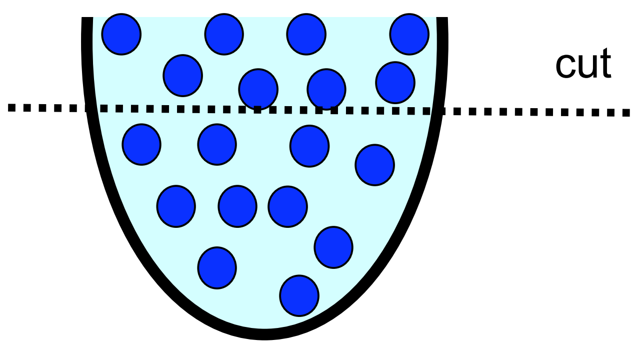
$$k \cot \delta = -\frac{1}{a}$$

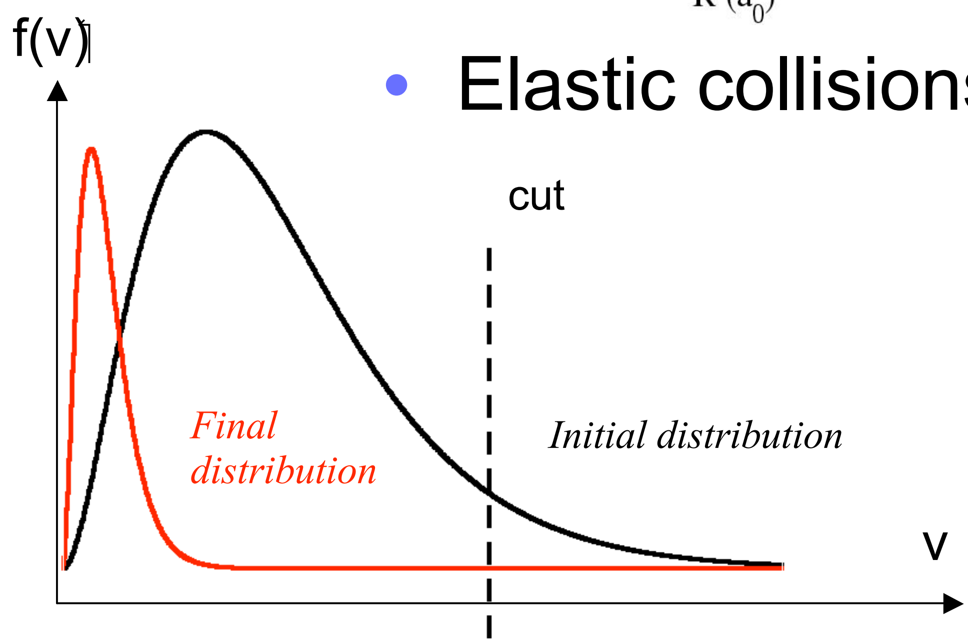
Scattering length a



- Elastic collisions

Evaporative cooling





Effective range expansion

$$k^{2\ell+1}\cot\delta_\ell(k) = c_0 + c_1 k^2 + c_2 k^4 + \cdots$$

- For s-wave and various power-law potentials

$$V = -\frac{C_3}{r^3} \qquad k\cot\delta_0 = -\frac{1}{a\ln k} +$$

$$V \sim -\alpha Q^2/2r^4 \qquad P^2 = \alpha m Q^2/\hbar^2 \qquad k\cot\delta_0 = -\frac{1}{a} + \frac{\pi P^2}{3a^2}k + \frac{4P^2}{3a}k^2\ln\left(\frac{Pk}{4}\right) +$$

$$V(r) = -\frac{C_6}{r^6} \qquad k\cot\delta_0 = -\frac{1}{a} + \frac{1}{2}r_e k^2 - \frac{2\pi}{15a^2}\frac{\mu C_6}{\hbar^2}k^3 +$$

$$V(r) = -\frac{C_7}{r^7} \qquad k\cot\delta_0 = -\frac{1}{a} + \frac{1}{2}r_e k^2 + \frac{4}{15a^2}\frac{\mu C_7}{\hbar^2}k^4\ln k +$$

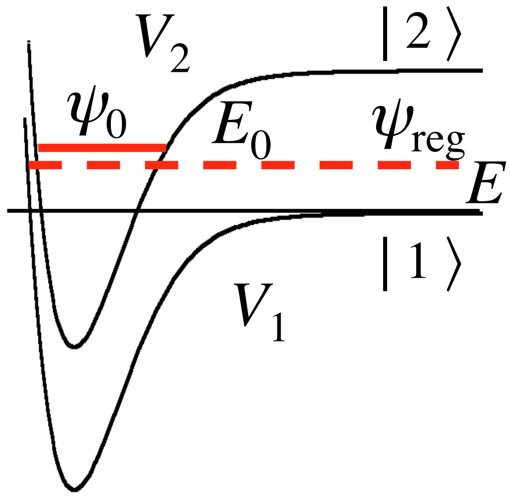
Feshbach resonance

- 2 channels: 1 open + 1 closed

$$|\Psi_{\rm tot}\rangle = \psi_1|1\rangle + \psi_2|2\rangle$$

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + \mathbf{V}$$

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} + \begin{pmatrix} V_1 & V_{1,2}\\ V_{2,1} & V_2\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = E\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$



- solutions

$$\begin{aligned}\psi_1(R) &= \psi_{\rm reg}(R) + \tan\delta\,\psi_{\rm irr}(R)\ ,\\ &\overset{R\to\infty}{=} \frac{1}{\cos\delta}\sqrt{\frac{2\mu}{\pi\hbar^2 k}}\sin(kR+\delta_{\rm bg}+\delta)\ ,\\ \psi_2(R) &= -\sqrt{\frac{2}{\pi\Gamma}}\sin\delta\,\psi_0(R)\end{aligned}$$

Resonance

- Resonant phase shift

$$\tan\delta = -\pi \frac{\overbrace{|\langle\psi_0|V_{2,1}|\psi_{\text{reg}}\rangle|^2}^{\Gamma/2\pi}}{E - \underbrace{E_0 - \langle\psi_0|V_{2,1}GV_{1,2}|\psi_0\rangle}_{E_R}}$$

$$G(R,R') = \begin{cases} \psi_{\text{reg}}(R)\psi_{\text{irr}}(R') & \text{for } R \le R' \\ \psi_{\text{reg}}(R')\psi_{\text{irr}}(R) & \text{for } R' \le R \end{cases}$$

$$\psi_{\text{reg}}(R) \overset{R\to\infty}{\longrightarrow} \sqrt{\frac{2\mu}{\pi\hbar^2 k}} \sin(kR + \delta_{\text{bg}})$$

$$\psi_{\text{irr}}(R) \overset{R\to\infty}{\longrightarrow} \sqrt{\frac{2\mu}{\pi\hbar^2 k}} \cos(kR + \delta_{\text{bg}})$$

$$\tan\delta = -\frac{\Gamma/2}{E - E_R}$$

Scattering length

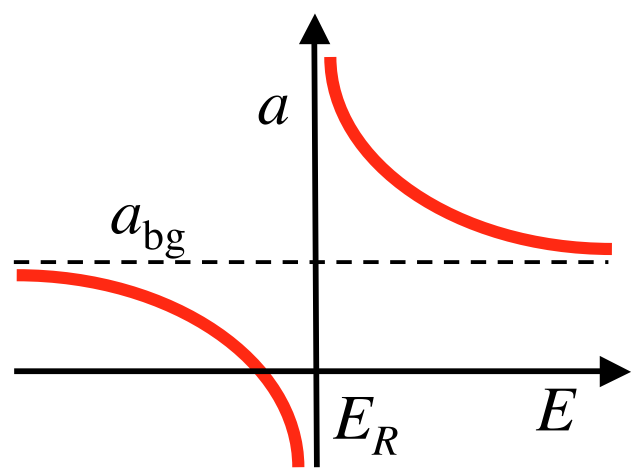
- As before $\psi_1 \sim \sin(kR + \delta + \delta_{\mathrm{bg}}) = \sin(kR + \delta_{\mathrm{tot}})$

$$\tan \delta_{\mathrm{tot}} = \tan(\delta + \delta_{\mathrm{bg}}) = -ka$$

$$-\frac{\Gamma/2}{E - E_R} \qquad\qquad -ka_{\mathrm{bg}}$$

$$-ka = \frac{\tan \delta + \tan \delta_{\mathrm{bg}}}{1 - \tan \delta_{\mathrm{bg}} \tan \delta}$$

$$a = a_{\mathrm{bg}} \left(1 + \frac{\Gamma/2}{k a_{\mathrm{bg}} (E - E_R)} \right)$$

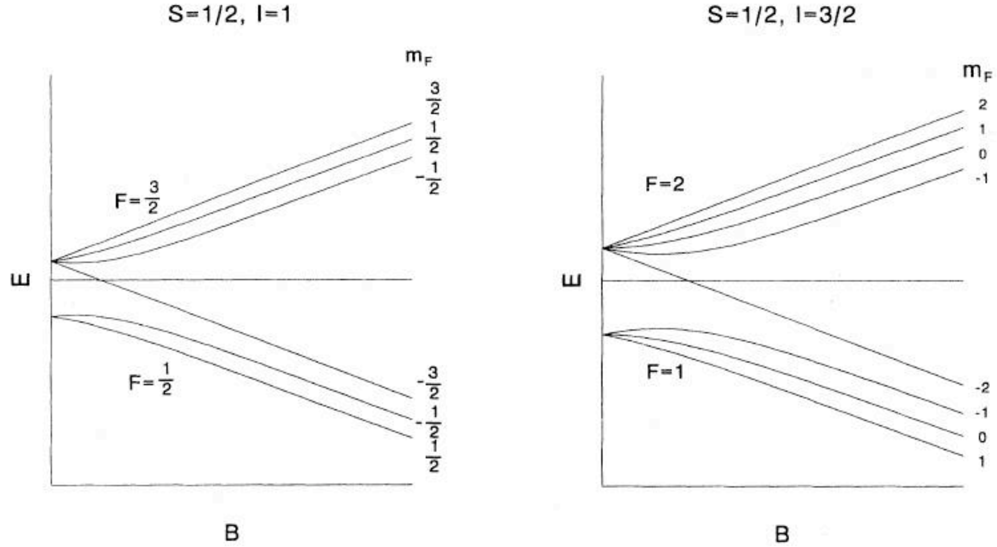


Real systems

- Hamiltonian: Zeeman and hyperfine couplings
- Ex: s-wave collisions ^{6}Li + ^{23}Na

$$H = \frac{p^2}{2\mu} + V_C + \sum_{j=1}^{2} H_j^{\text{int}}$$

$$V_C = V_S(R)P^S + V_T(R)P^T$$



$$H_j^{\text{int}} = \frac{a_{\text{hf}}^{(j)}}{\hbar^2}\mathbf{s}_j \cdot \mathbf{i}_j + (\gamma_e \mathbf{s}_j - \gamma_n \mathbf{i}_j) \cdot \mathbf{B}$$

^{6}Li: 152.137 MHz
^{23}Na: 885.813 MHz

^{6}Li $\qquad$ ^{23}Na

$$|\Psi_{\epsilon,\ell=0}\rangle = \sum_{\sigma=1}^{N} \psi_\sigma(R)\{|f_1, m_1\rangle \otimes |f_2, m_2\rangle\}_\sigma \qquad \mathbf{f}_j = \mathbf{i}_j + \mathbf{s}_j$$

Magnetically tuned

- Instead of changing E

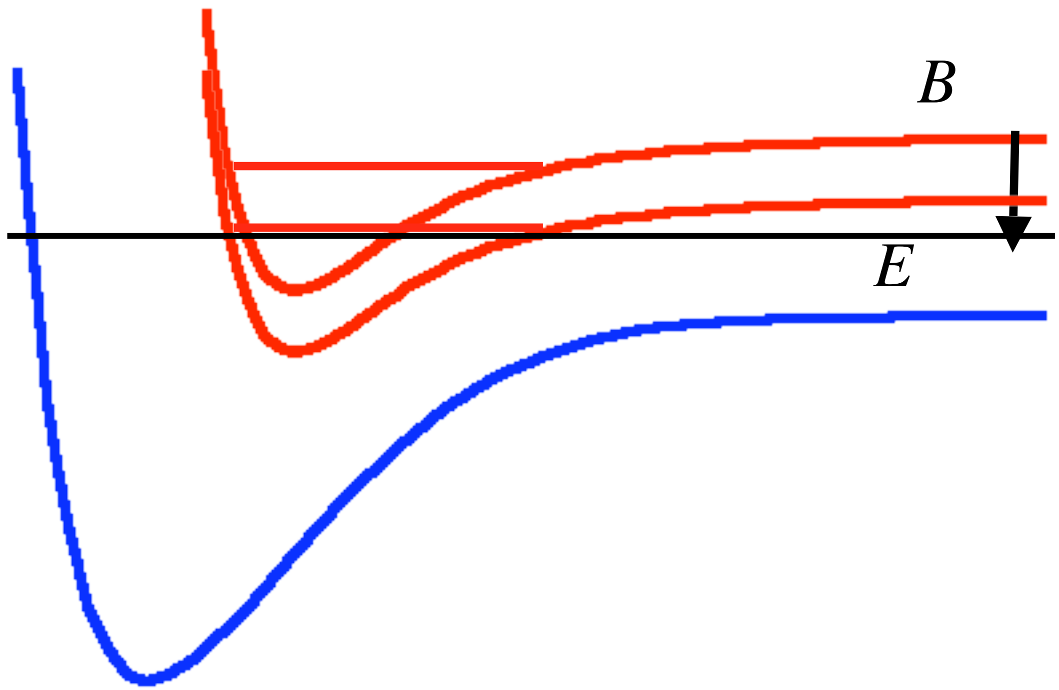
$$a = a_{\rm bg}\left(1 + \frac{\Gamma/2}{k a_{\rm bg}(E - E_R)}\right)$$

  – Zeeman term moves curves with respect to each other

$$a = a_{\rm bg}\left(1 - \frac{\Delta}{B - B_0}\right)$$

Δ linked to Γ

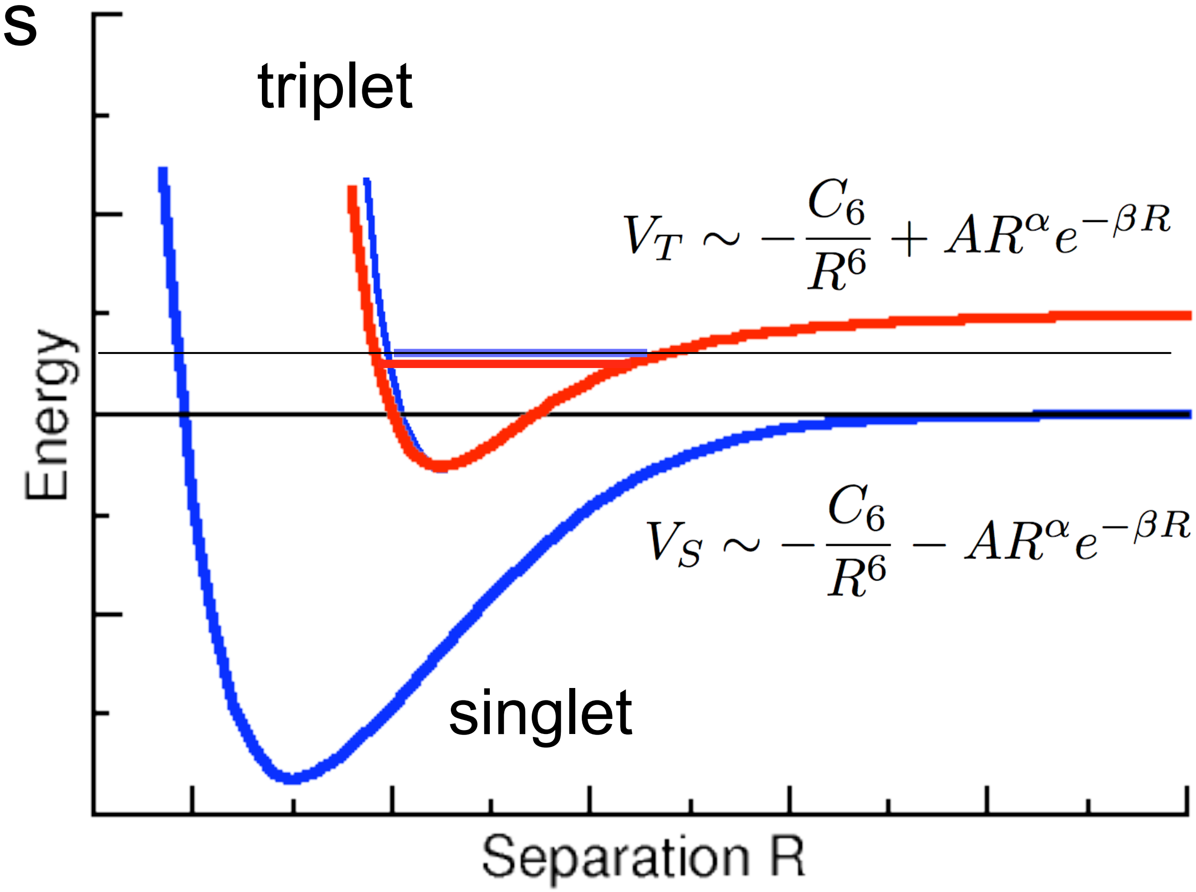
B_0 linked to E_R



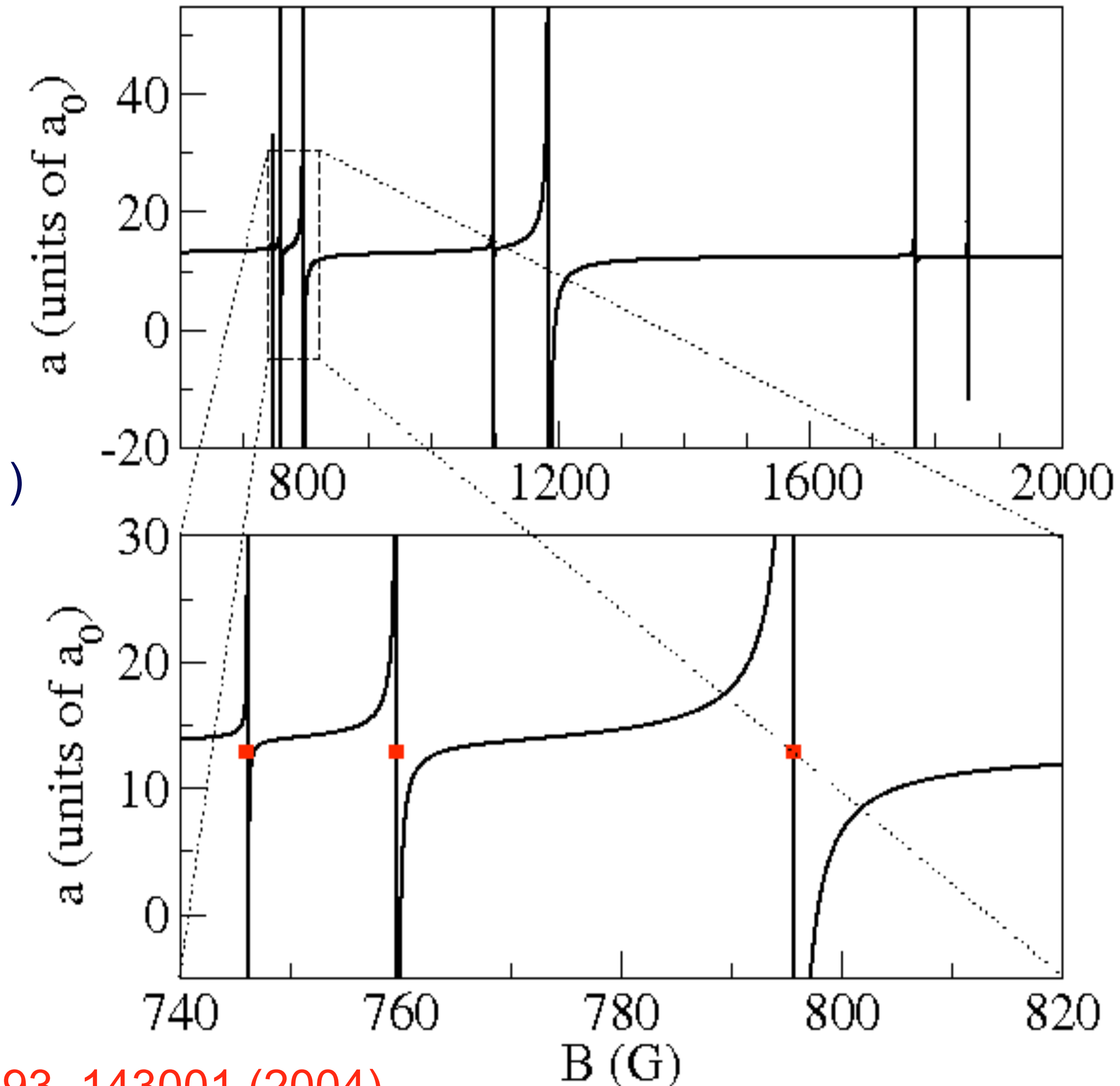
Adjusting potentials

- Adjust inner wall
  – Move bound state up or down

- Other approaches
  – Adjust singlet or triplet phase



Results with Li+Na



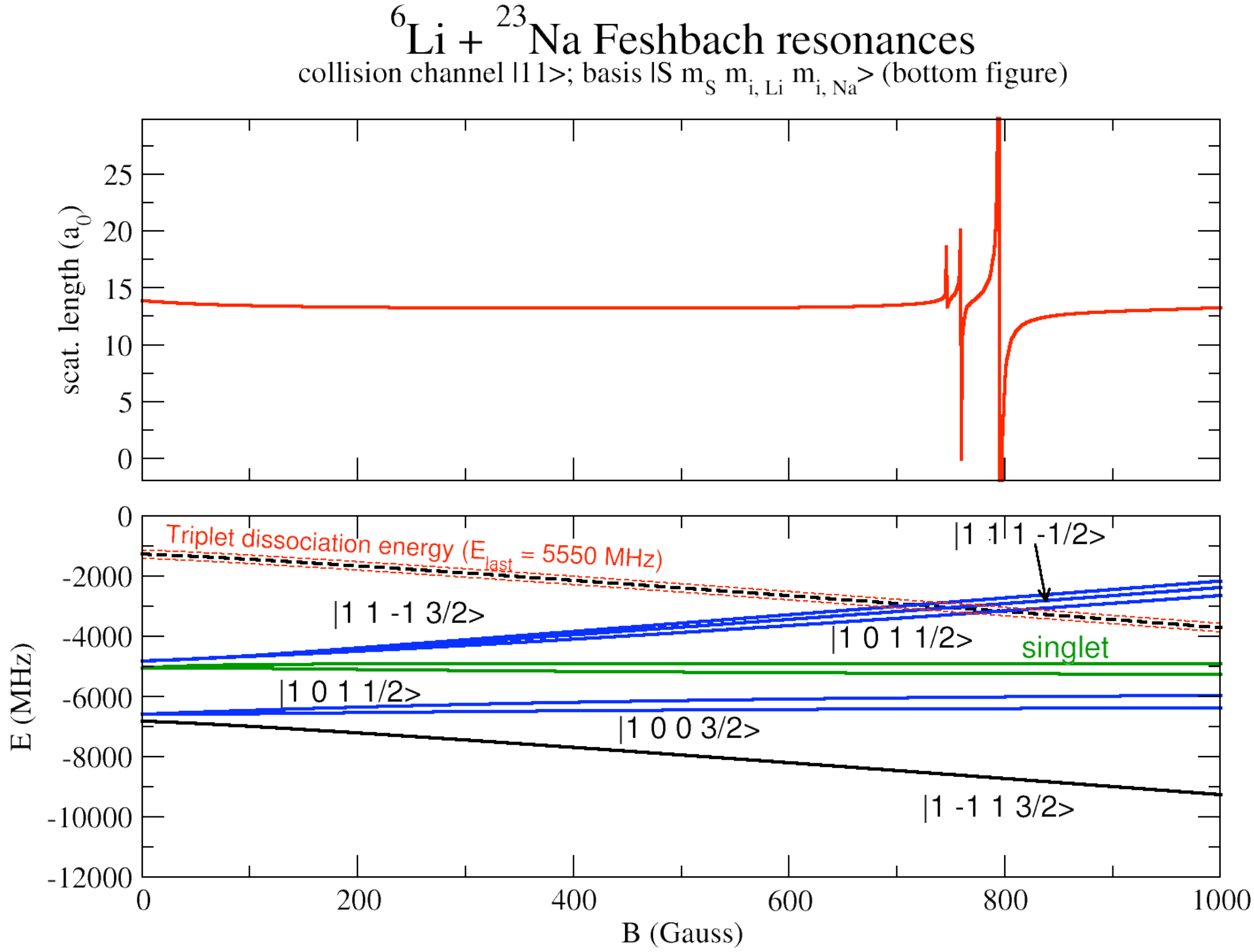
^{6}Li(f =1/2, m_f = 1/2) ^{23}Na(f=1, m_f =1)

Theory B (Gauss)	Experiment * B (Gauss)
746.13	746.0 +/- 0.4
759.69	759.6 +/- 0.2
795.61	795.6 +/-0.2

C. A. Stan et al, Phys. Rev. Lett.,93, 143001 (2004)
M. Gacesa et al.,PRA 78, 010701 (R) (2008).

Bound state + continuum

- Resonances vs bound state



Coffee Break !

- Ultracold atoms after the 1st break