Dipolar Fermi Gas:

Novel Properties and Open Questions

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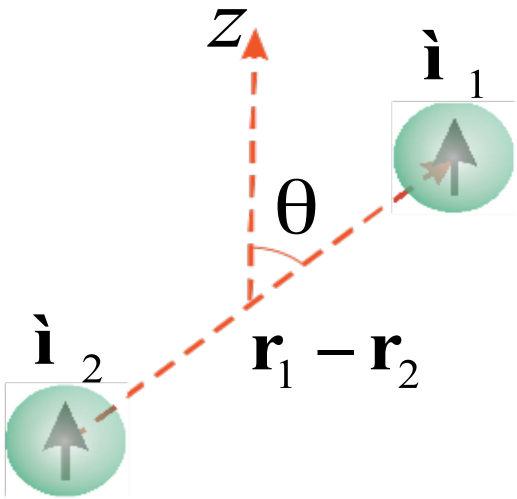
Dipolar interaction between two atoms

$$V_d(\mathbf{r}_1-\mathbf{r}_2)=\frac{\mu_0}{4\pi}\frac{\mu_1\cdot\mu_2-3(\mu_1\cdot\hat{\mathbf{r}})(\mu_2\cdot\hat{\mathbf{r}})}{|\mathbf{r}_1-\mathbf{r}_2|^3}$$

$$=d^2\frac{1-3\cos^2\theta}{|\mathbf{r}_1-\mathbf{r}_2|^3}\quad\text{(Polarized dipole)}$$



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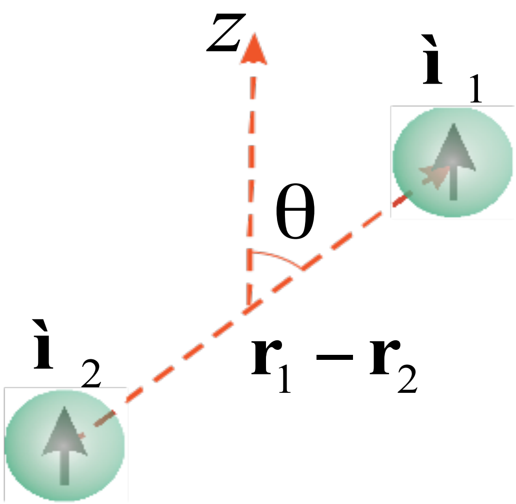

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$$=d^2\frac{1-3\cos^2\theta}{|\mathbf{r}_1-\mathbf{r}_2|^3}$$ (Polarized dipole)

$$\tilde{V}(\mathbf{k})=\frac{4\pi d^2}{3}(3\cos^2\theta_{\mathbf{k}}-1)$$

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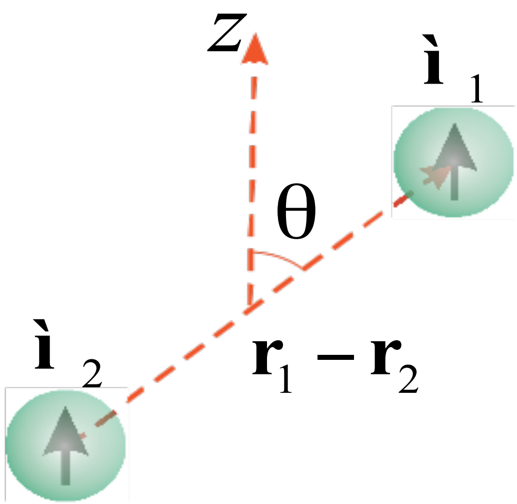

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$$= d^2 \frac{1 - 3\cos^2\theta}{|\mathbf{r}_1 - \mathbf{r}_2|^3}$$

(Polarized dipole)

$$\tilde{V}(\mathbf{k}) = \frac{4\pi d^2}{3}(3\cos^2\theta_{\mathbf{k}} - 1)$$

- Long ranged (~1/R^3)
- Anisotropic

Why can dipolar interaction become important?

For typical alkali BEC, the s-wave interaction energy ~ 100 nK
BEC transition temperature ~ 100 nK
the dipolar interaction energy ~ 0.1 nK

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Dipolar vs. finite temperature: Bose enhancement factor

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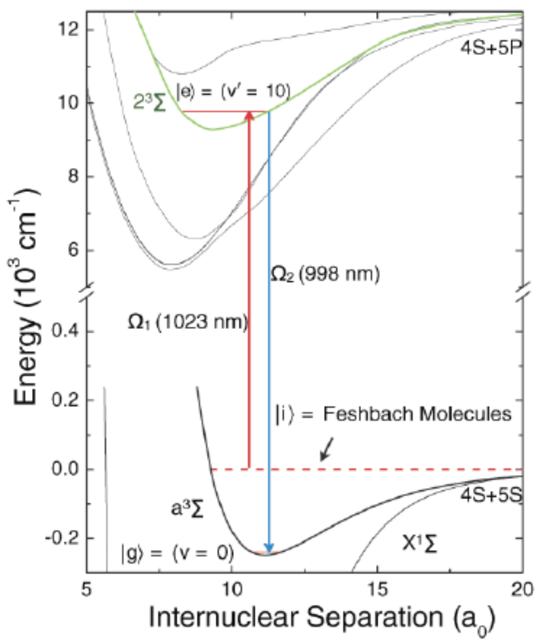
How about dipolar fermions?

Polar molecule as dipolar fermions

Science AAAS

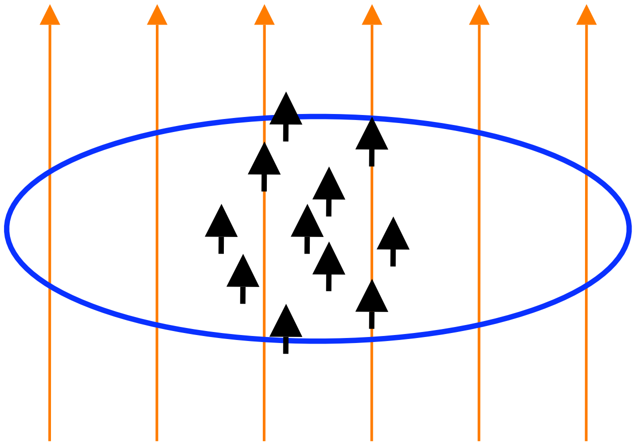
A High Phase-Space-Density Gas of Polar Molecules
K.-K. Ni, *et al.*
Science **322**, 231 (2008);
DOI: 10.1126/science.1163861



^{40}K ^{87}Rb

Large electric dipole moment: 0.57 Debye (singlet)
Dipolar interaction ~100 times larger than in ^{52}Cr

Model



N spin polarized (along z-axis) fermions
Interacting with each other via the
dipolar interaction

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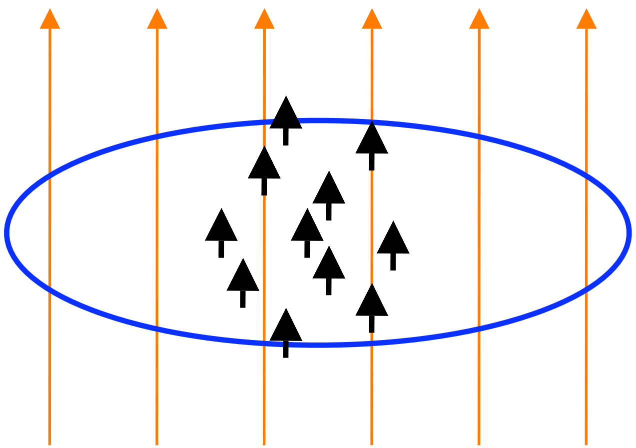

N spin polarized (along z-axis) fermions
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Hamiltonian

$$H = \sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\nabla_i^2 + U(\mathbf{r}_i)\right] + \frac{1}{2}\sum_{i\neq j} V(\mathbf{r}_i - \mathbf{r}_j)$$

$$V(\mathbf{r}_i - \mathbf{r}_j) = d^2 \frac{1 - 3\cos^2\theta}{|\mathbf{r}_i - \mathbf{r}_j|^3}$$

Semiclassical variational approach

PRA **77**, 061603(R) (2008)
New J. Phys. **11**, 055017 (2009)

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Wigner Distribution Function

$$\rho(\mathbf{r},\mathbf{r}')=\frac{1}{(2\pi)^3}\int d^3k\, f\left(\frac{\mathbf{r}+\mathbf{r}'}{2},\mathbf{k}\right)e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}$$

$$n(\mathbf{r})=\rho(\mathbf{r},\mathbf{r})=\frac{1}{(2\pi)^3}\int d^3k\, f(\mathbf{r},\mathbf{k})$$

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$$n(\mathbf{r}) = \rho(\mathbf{r}, \mathbf{r}) = \frac{1}{(2\pi)^3} \int d^3k\, f(\mathbf{r}, \mathbf{k})$$

$$\tilde{n}(\mathbf{k}) = \frac{1}{(2\pi)^3} \int d^3r\, f(\mathbf{r}, \mathbf{k})$$

Choose the proper f that minimizes the total energy

Total energy

$$E_{kin} = \int d^3k\ \tilde{n}(\mathbf{k}) \frac{\hbar^2 k^2}{2m}$$

$$E_{trap} = \int d^3r\ n(\mathbf{r}) U(\mathbf{r})$$

$$E_{Hartree} = \frac{1}{2} \iint d^3r d^3r'\ n(\mathbf{r}) V_d(\mathbf{r}-\mathbf{r}') n(\mathbf{r}')$$

$$E_{Fock} = -\frac{1}{2(2\pi)^6} \iint d^3r d^3r' \iint d^3k d^3k'\ V_d(\mathbf{r}-\mathbf{r}') e^{i(\mathbf{k}-\mathbf{k}')\cdot(\mathbf{r}-\mathbf{r}')} f\left(\frac{\mathbf{r}+\mathbf{r}'}{2}, \mathbf{k}\right) f\left(\frac{\mathbf{r}+\mathbf{r}'}{2}, \mathbf{k}'\right)$$

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Goal: minimize the total energy
Strategy: treat the Wigner function variationally

Homogeneous case: Wigner function

$$f(\mathbf{r},\mathbf{k}) = f(\mathbf{k}) = \Theta\left(k_F^2 - \tfrac{1}{\alpha}\left(k_x^2 + k_y^2\right) - \alpha^2 k_z^2\right)$$

$k_F = \left(6\pi^2 n_f\right)^{1/3}$: Fermi momentum for a noninteracting system

α : variational parameter, characterizing deformation of Fermi surface

$$\alpha \begin{cases} <1: & \text{prolate Fermi surface} \\ =1: & \text{isotropic Fermi surface} \\ >1: & \text{oblate Fermi surface} \end{cases}$$

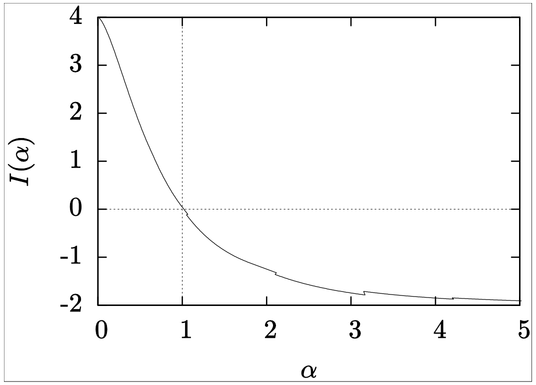
Homogeneous case: energies

$$E_{trap} = 0$$

$$E_{Hartree} = 0$$

$$E_{kin} = \frac{V}{5}\frac{\hbar^2 k_F^2}{2m} n_f \left(\alpha^{-2} + 2\alpha\right)$$

$$E_{Fock} = -\frac{\pi d^2 V}{3} n_f^2 I(\alpha)$$



$$I(\alpha) = \int_0^\pi d\theta \ \sin\theta \left(\frac{3\cos^2\theta}{\alpha^3 \sin^2\theta + \cos^2\theta} - 1\right)$$

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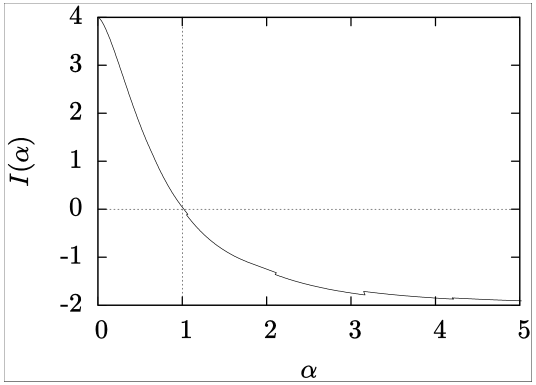

Kinetic energy favors an isotropic Fermi surface ($\alpha = 1$);

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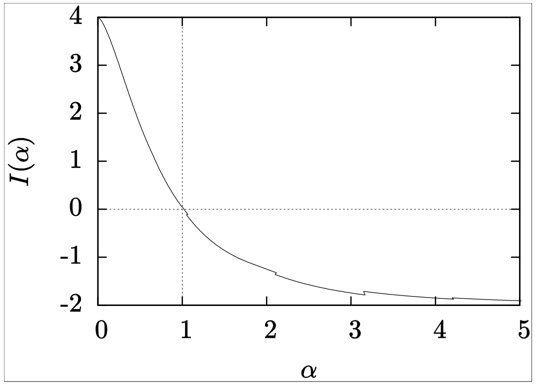

Kinetic energy favors an isotropic Fermi surface (α =1);
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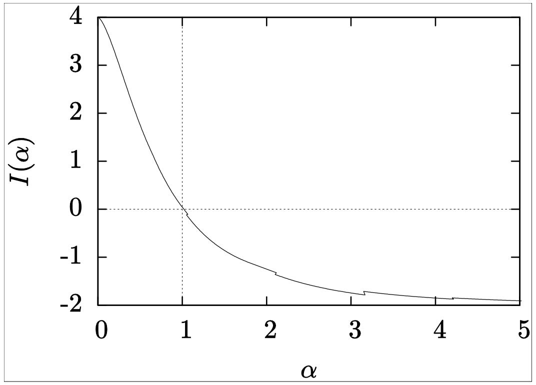

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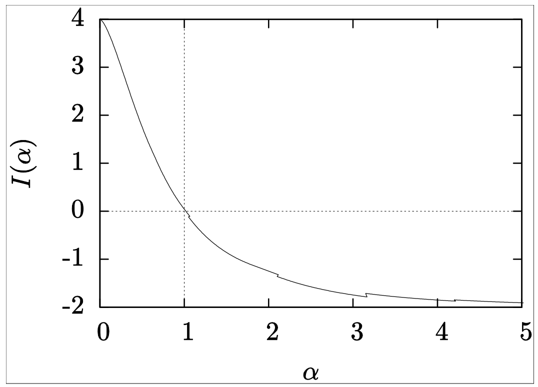

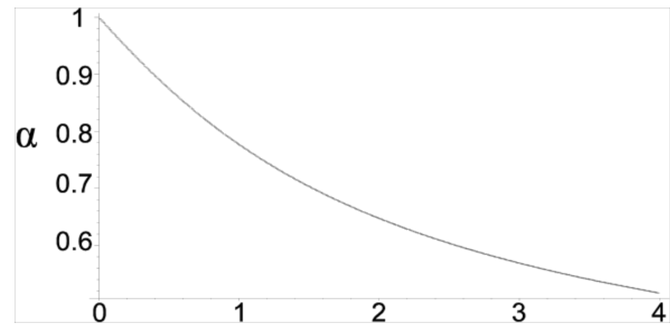

$$C_{dd} = \frac{m}{\hbar^2} n_f^{1/3} d^2$$

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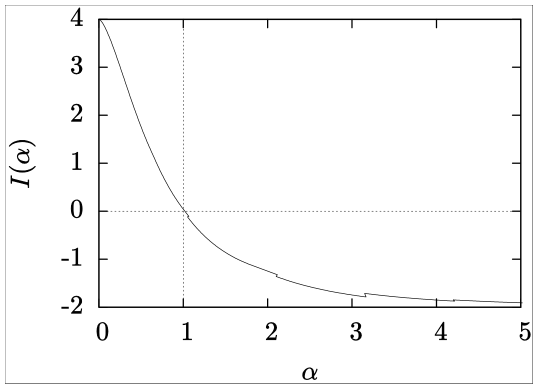

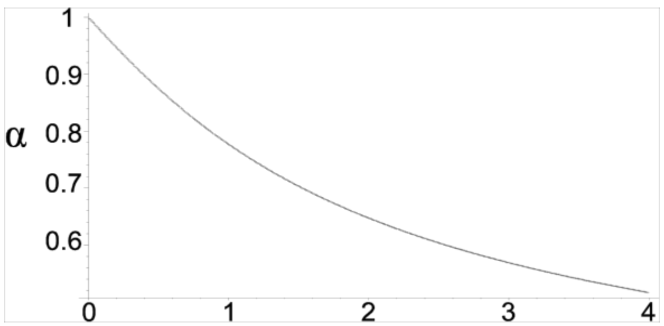

$$C_{dd} = \frac{m}{\hbar^2} n_f^{1/3} d^2$$

$$\mathrm{KRb}@10^{13}\,cm^{-3},\ C_{dd} \approx 1.32$$

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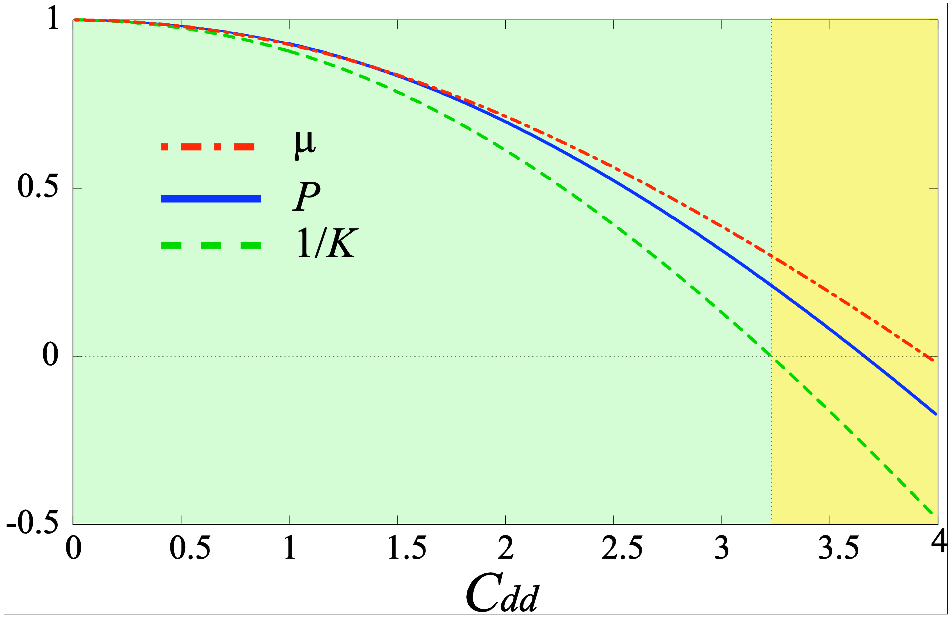
Homogeneous case: stability

chemical potential : μ

pressure : P

bulk modulus : $1/K$



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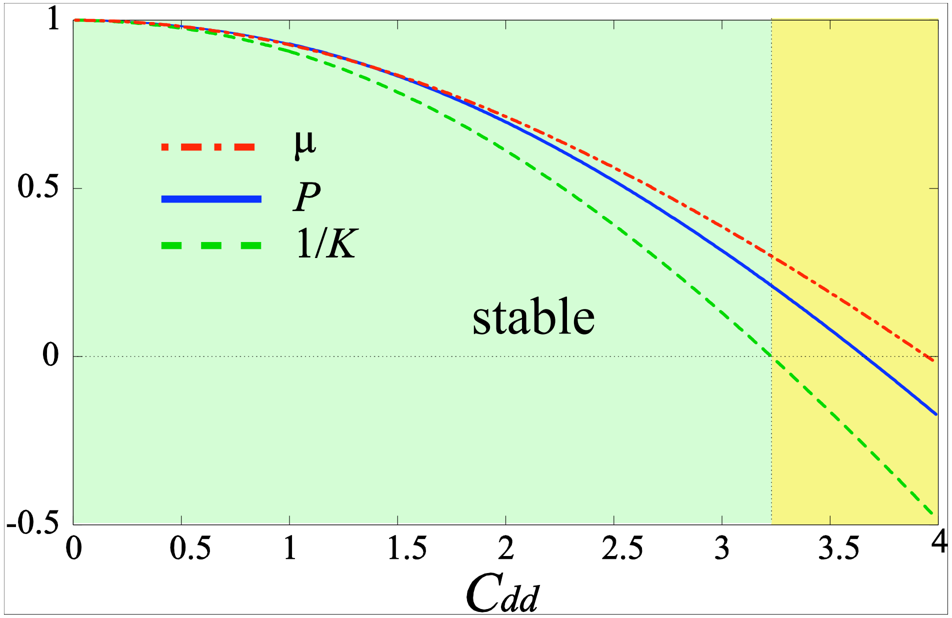

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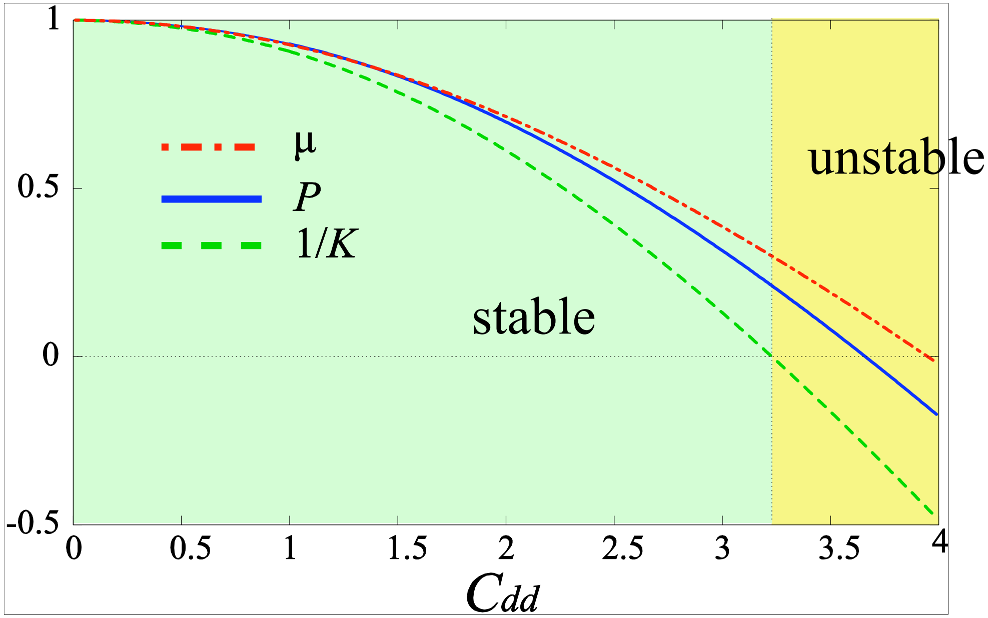

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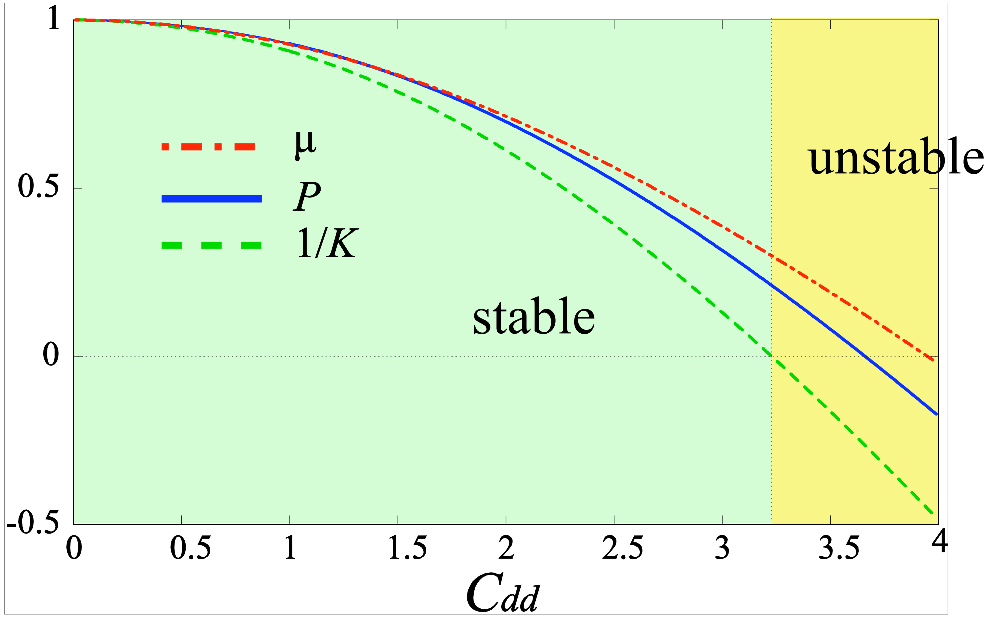

Sufficiently large dipolar strength leads to collapse.

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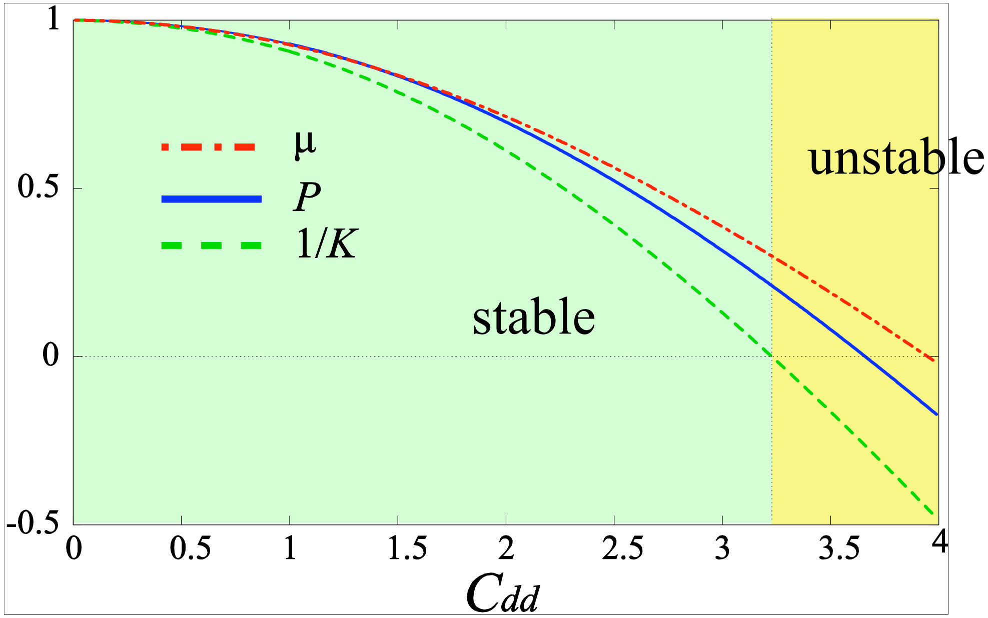

Sufficiently large dipolar strength leads to collapse.

$1\,\text{Debye}, 100\,\text{a.m.u.}, 10^{13}\,cm^{-3},\;\; C_{dd} \approx 3.2$

Inhomogeneous case: Wigner function

$$U(\mathbf{r}) = \tfrac{m}{2}\left[\omega_r^2\left(x^2+y^2\right)+\omega_z^2 z^2\right]$$

$$f(\mathbf{r},\mathbf{k}) = \Theta\left(k_F^2(\mathbf{r}) - \tfrac{1}{\alpha}\left(k_x^2+k_y^2\right) - \alpha^2 k_z^2\right)$$

$$k_F^2(\mathbf{r}) = k_F^{0\,2} - \lambda^2\left[\beta\left(x^2+y^2\right)+\beta^{-2}z^2\right]$$

$$\frac{1}{(2\pi)^6}\iint d^3r\, d^3k\, f(\mathbf{r},\mathbf{k}) = N \Rightarrow k_F^0 = (48N)^{1/6}\lambda^{1/2}$$

β : variational parameter, characterizing deformation in real space

λ : variational parameter, characterizing scaling factor

($\lambda > 1$, shrinking in real space)

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($\lambda > 1$, shrinking in real space)

Similar treatment by Goral *et al*. in PRA **63**, 033606 (2001), but with α=1.

Inhomogeneous case: energies

$$E_{trap} = c_1 N^{4/3} \lambda^{-1} \left(2\frac{\beta_0}{\beta} + \frac{\beta^2}{\beta_0^2} \right), \qquad \beta_0 \equiv \left(\frac{\omega_r}{\omega_z} \right)^{2/3} : \text{trap aspect ratio}$$

$$E_{kin} = c_1 N^{4/3} \lambda \left(\alpha^{-2} + 2\alpha \right)$$

$$E_{Hartree} = c_2 N^{3/2} \lambda^{3/2} d^2 \; I(\beta)$$

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Interaction energy is not bounded from below (dipolar interaction is partially attractive).
The system is not absolutely stable against collapse ($\lambda \to \infty$).

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A local minimum may exist: the system may sustain a metastable state.

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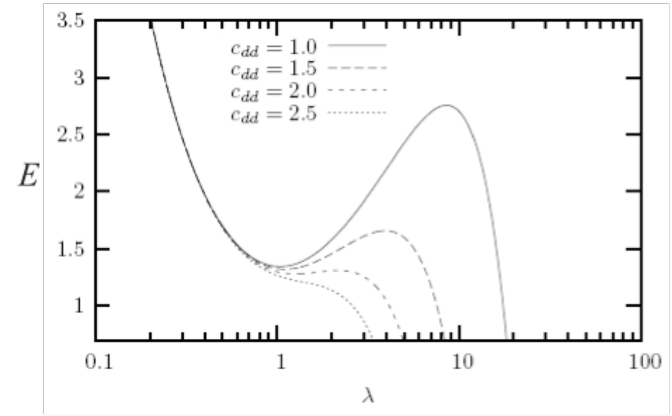

$$C_{dd} = \frac{N^{1/6} d^2}{a_{ho}^3 \hbar\omega}$$

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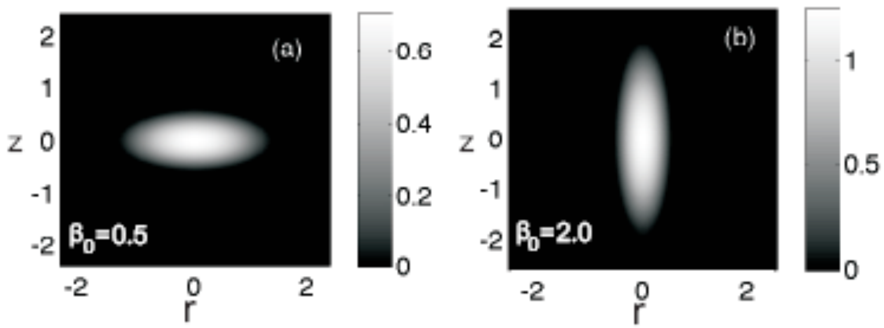
Density profiles

Real space: $C_{dd} = 1.5$



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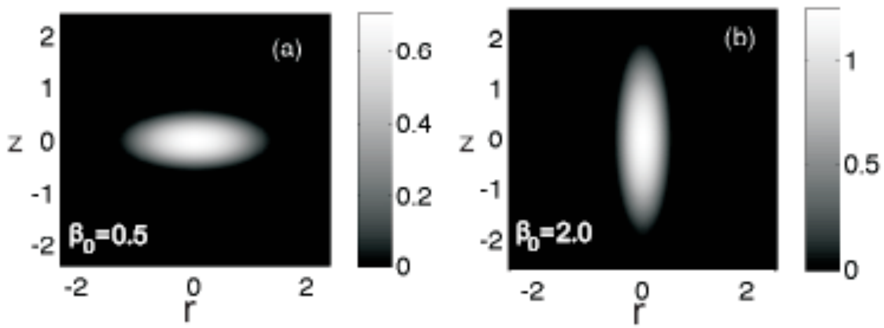
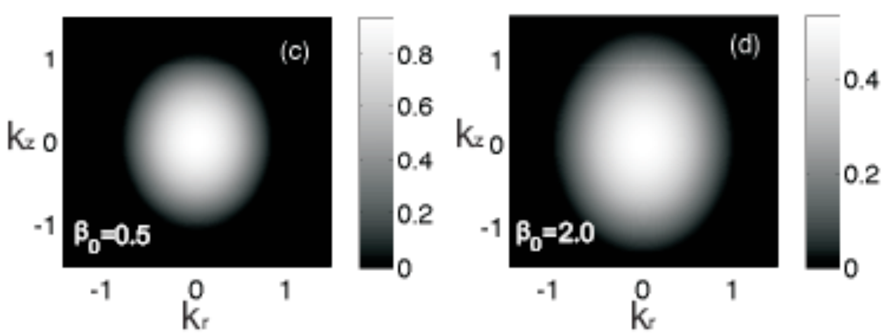


Momentum space:



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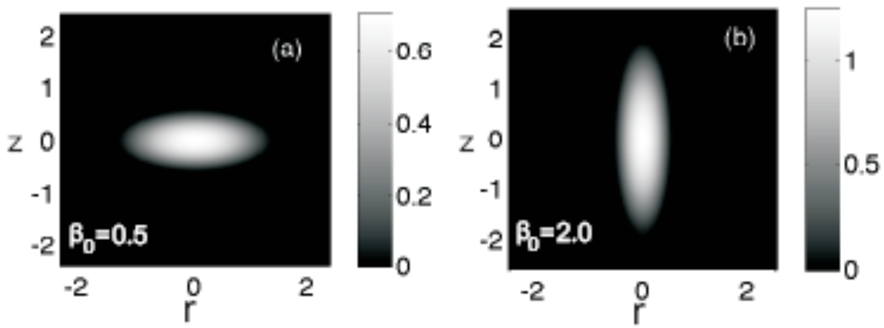
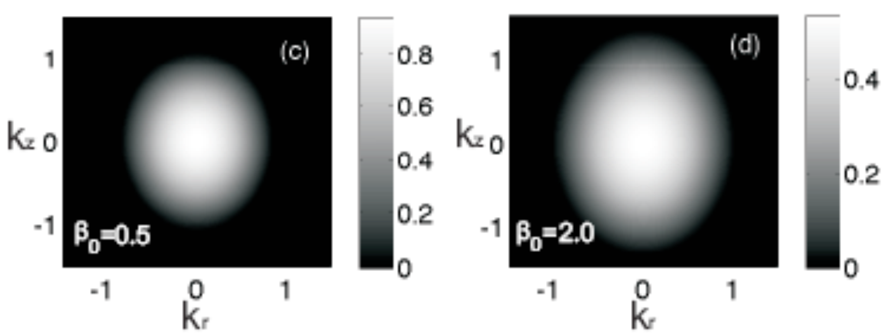
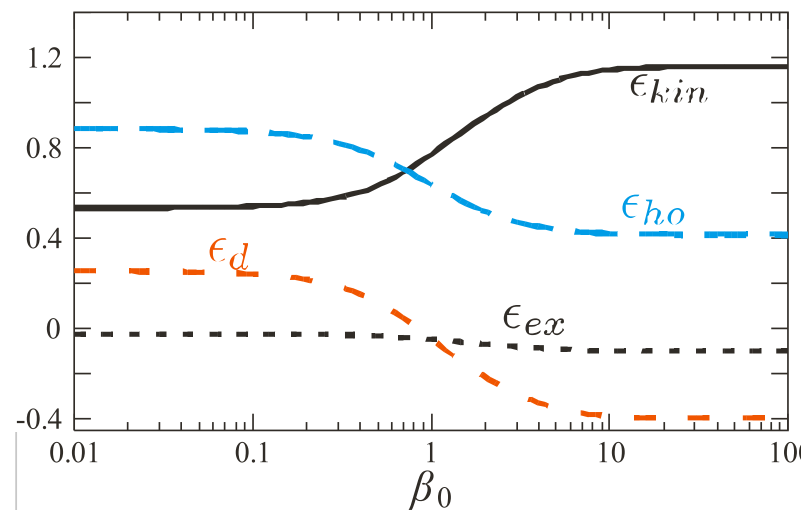


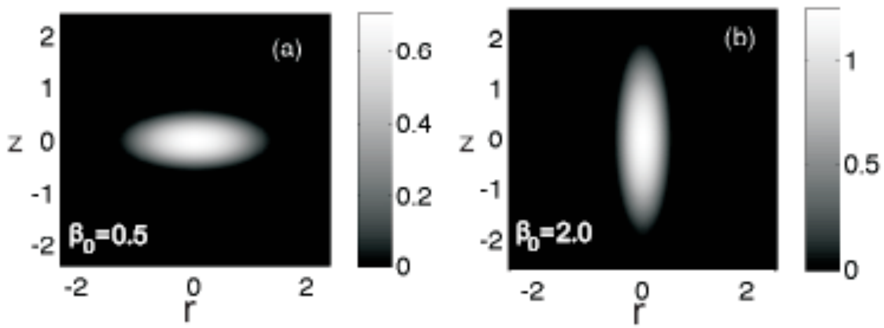
Momentum space:





Density profiles

Real space: $C_{dd} = 1.5$



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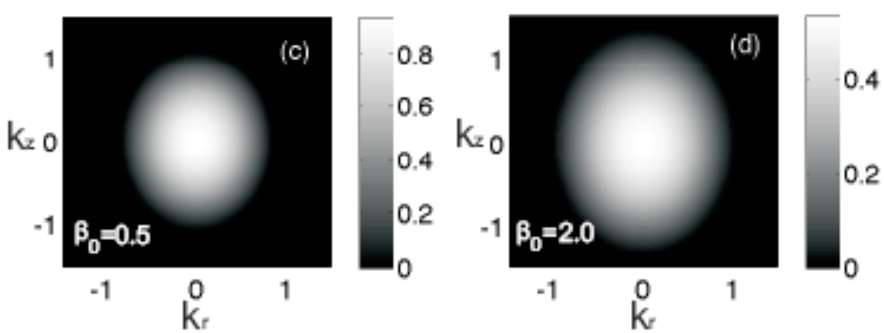

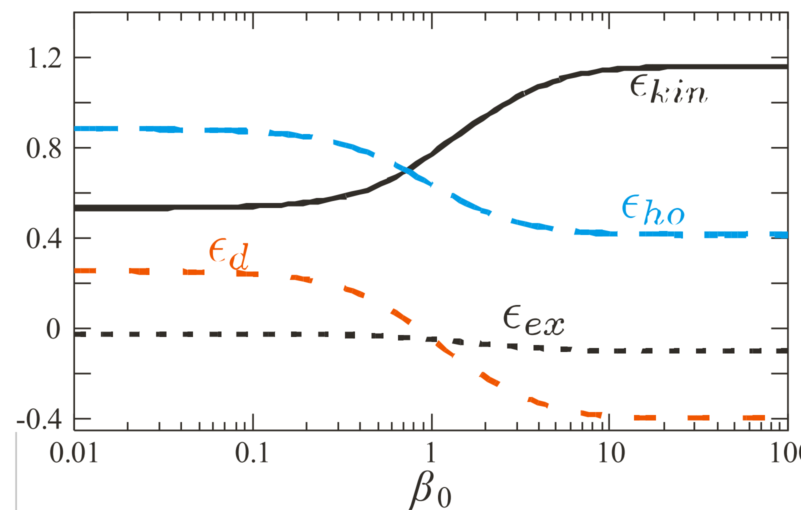

KRb@ $N = 10^6$, $\omega = 2\pi \times 100\text{Hz}$, $C_{dd} \approx 7.5$

Hartree-Fock-Bogoliubov Thoery

Hartree-Fock-Bogoliubov Thoery

$$H = \sum_{\mathbf{k}} \left(\varepsilon_k - \mu\right) a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + \tfrac{1}{2} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V_{\mathbf{k},\mathbf{k}',\mathbf{k}+\mathbf{q},\mathbf{k}+\mathbf{q}}\, a^\dagger_{\mathbf{k}} a^\dagger_{\mathbf{k}'} a_{\mathbf{k}'-\mathbf{q}} a_{\mathbf{k}+\mathbf{q}}$$

$$V_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4} = \tfrac{1}{\Omega^2} \iint d\mathbf{x}_1 d\mathbf{x}_2 \; e^{-i(\mathbf{k}_1\cdot\mathbf{x}_1 + \mathbf{k}_2\cdot\mathbf{x}_2)} \; V(\mathbf{x}_1 - \mathbf{x}_2) \; e^{i(\mathbf{k}_3\cdot\mathbf{x}_1 + \mathbf{k}_4\cdot\mathbf{x}_2)}$$

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Bogoliubov transformation :

$$a_{\mathbf{k}} = u_{\mathbf{k}} \alpha_{\mathbf{k}} + \mathrm{sgn}(\mathbf{k}) v_{\mathbf{k}} \alpha^{\dagger}_{-\mathbf{k}}, \qquad \mathrm{sgn}(\mathbf{k}) = \mathrm{sgn}(k_z)$$

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$$H = \sum_{\mathbf{k}} \left[\left(\xi_{\mathbf{k}} - U_{HF}(\mathbf{k})\right) v_{\mathbf{k}}^2 + \tfrac{1}{2}\mathrm{sgn}(\mathbf{k})\Delta_{\mathbf{k}} u_{\mathbf{k}} v_{\mathbf{k}}\right] + \sum_{\mathbf{k}} \left[\xi_{\mathbf{k}}\left(u_{\mathbf{k}}^2 - v_{\mathbf{k}}^2\right) - 2\,\mathrm{sgn}(\mathbf{k})\Delta_{\mathbf{k}} u_{\mathbf{k}} v_{\mathbf{k}}\right]\alpha_{\mathbf{k}}^{\dagger}\alpha_{\mathbf{k}}$$

$$+ \sum_{\mathbf{k}} \left[\mathrm{sgn}(\mathbf{k})\xi_{\mathbf{k}} u_{\mathbf{k}} v_{\mathbf{k}} - \tfrac{1}{2}\Delta_{\mathbf{k}}\left(u_{\mathbf{k}}^2 - v_{\mathbf{k}}^2\right)\right]\left(\alpha_{-\mathbf{k}}\alpha_{\mathbf{k}} + \alpha_{\mathbf{k}}^{\dagger}\alpha_{-\mathbf{k}}\right)$$

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$$\Delta_{\mathbf{k}} = \sum_{\mathbf{k}'} \mathrm{sgn}(\mathbf{k}') V_{\mathbf{k},-\mathbf{k},\mathbf{k}',-\mathbf{k}'} u_{\mathbf{k}'} v_{\mathbf{k}'}$$

$$\xi_{\mathbf{k}} = \varepsilon_k - \mu + 2U_{HF}(\mathbf{k})$$

Self-consistent solution

Self-consistent solution

$$H = \sum_{\mathbf{k}} \left[\left(\xi_{\mathbf{k}} - U_{HF}(\mathbf{k}) \right) v_{\mathbf{k}}^{2} - \frac{\Delta_{\mathbf{k}}^{2}}{4E_{\mathbf{k}}} \right] + \sum_{\mathbf{k}} E_{\mathbf{k}} \alpha_{\mathbf{k}}^{\dagger} \alpha_{\mathbf{k}}$$

$$E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^{2} + \Delta_{\mathbf{k}}^{2}}, \quad \xi_{\mathbf{k}} = \varepsilon_{k} - \mu + 2U_{HF}(\mathbf{k}), \quad v_{\mathbf{k}}^{2} = \frac{1}{2}\left(1 - \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}} \right)$$

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Renormalization of gap equation:
Baranov et al., PRA 66, 013606 (2002)

Normal state

$$\Delta_{\mathbf{k}} = 0$$

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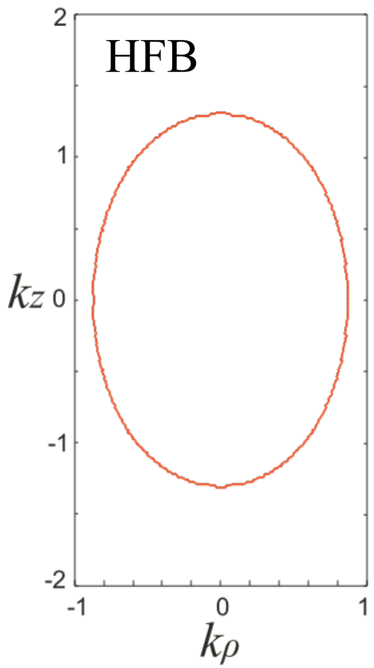
C_{dd}=1.096



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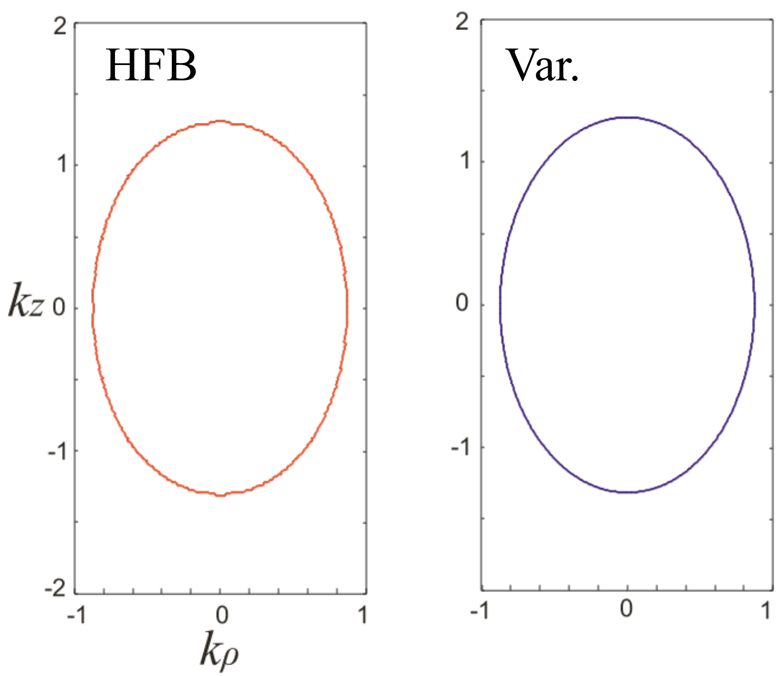

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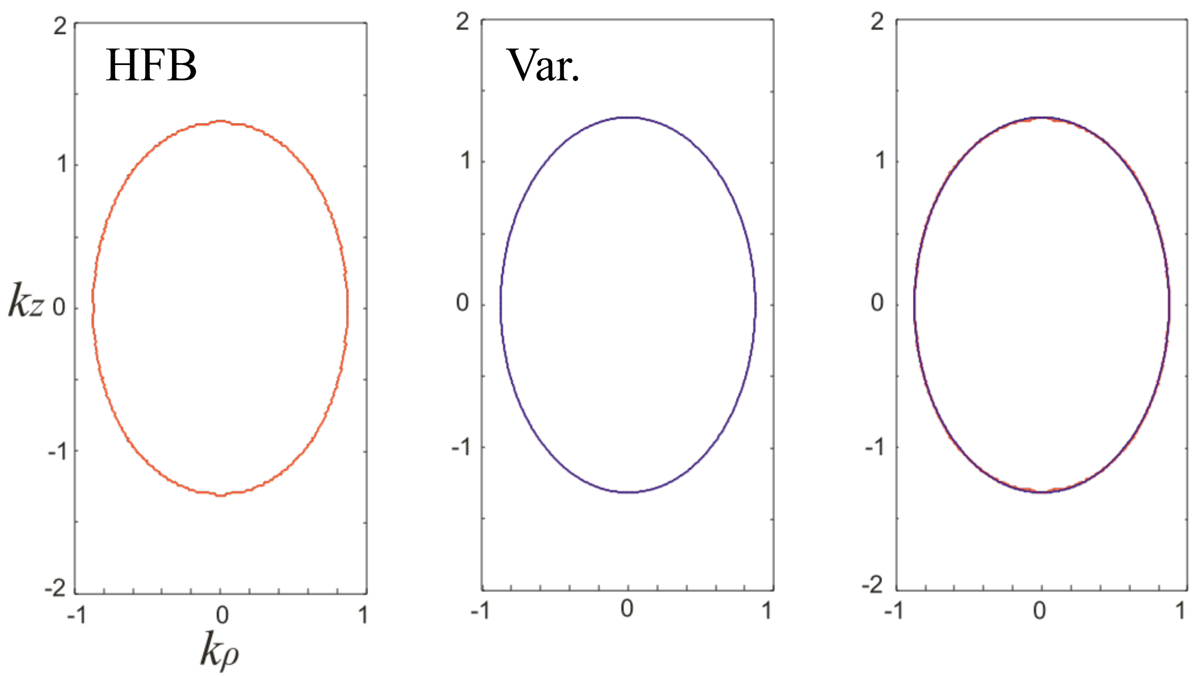

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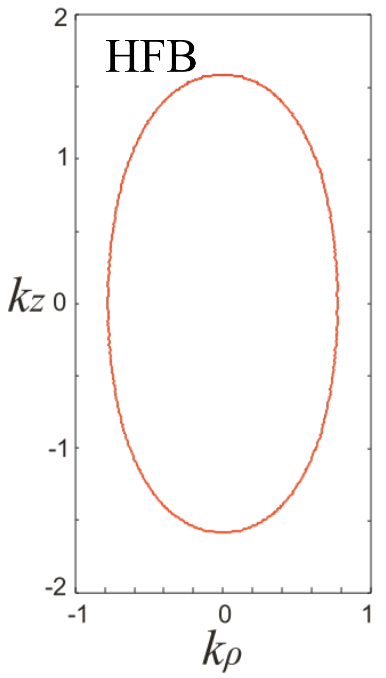

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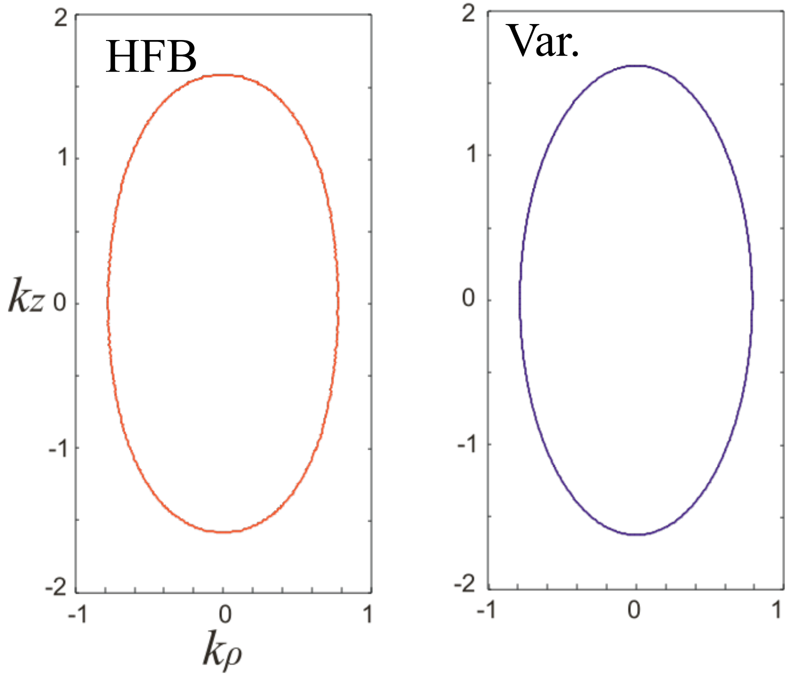

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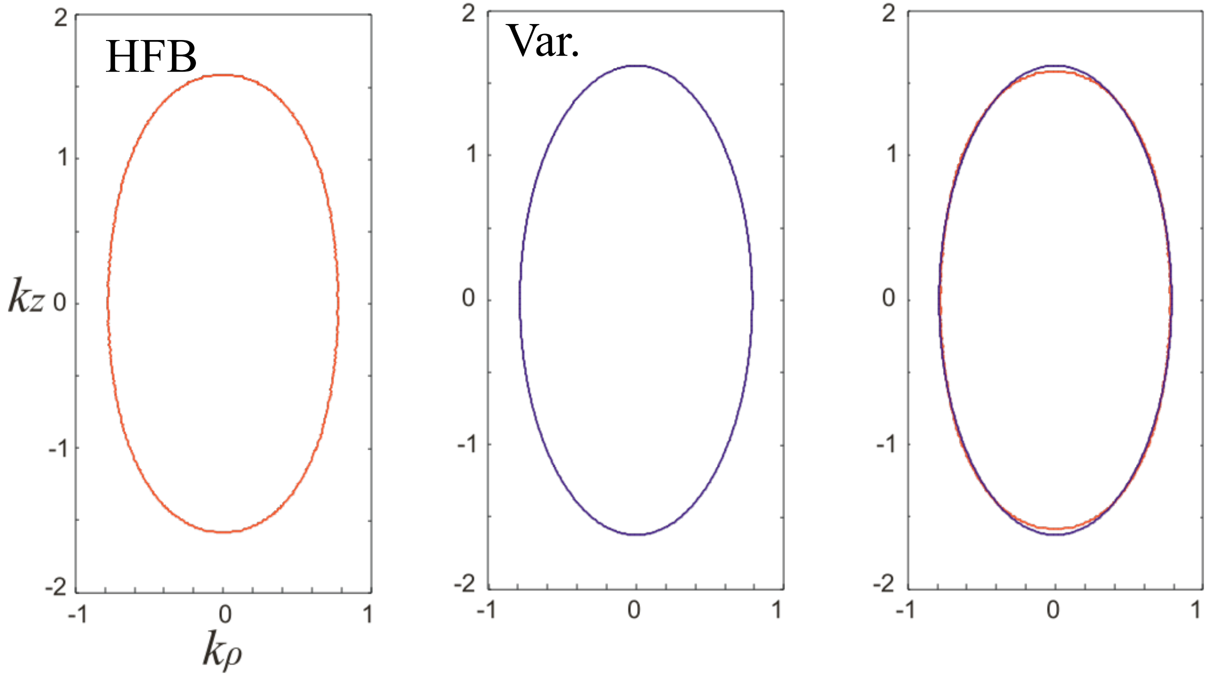


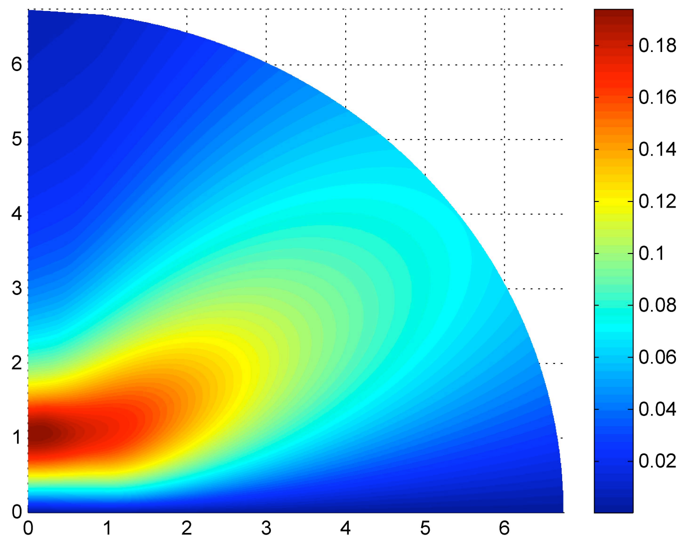
Superfluid state

$$C_{dd} = 1.0$$

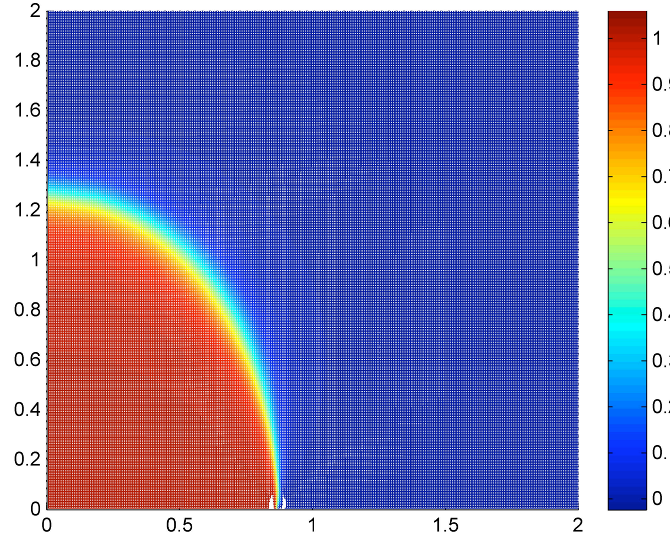
Order parameter

Momentum distribution

k_z



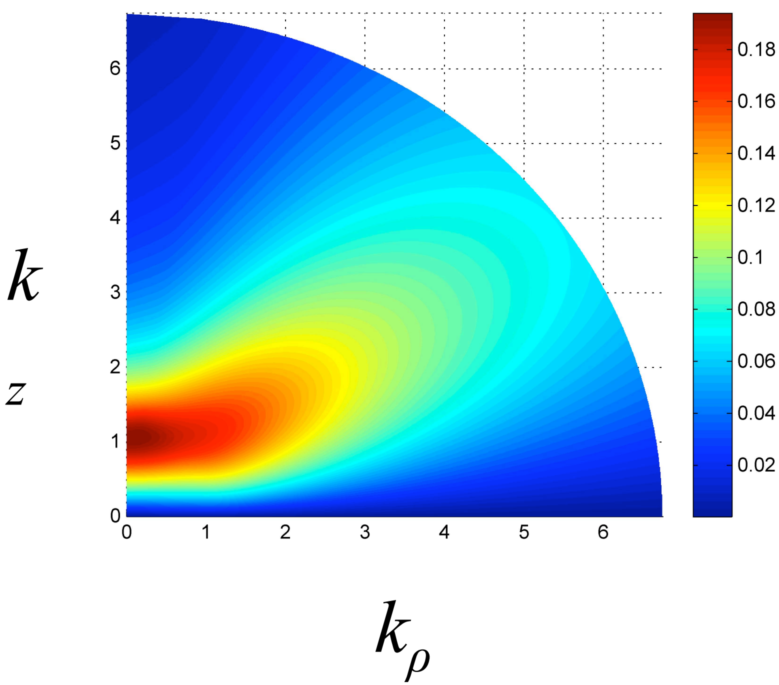
k_ρ

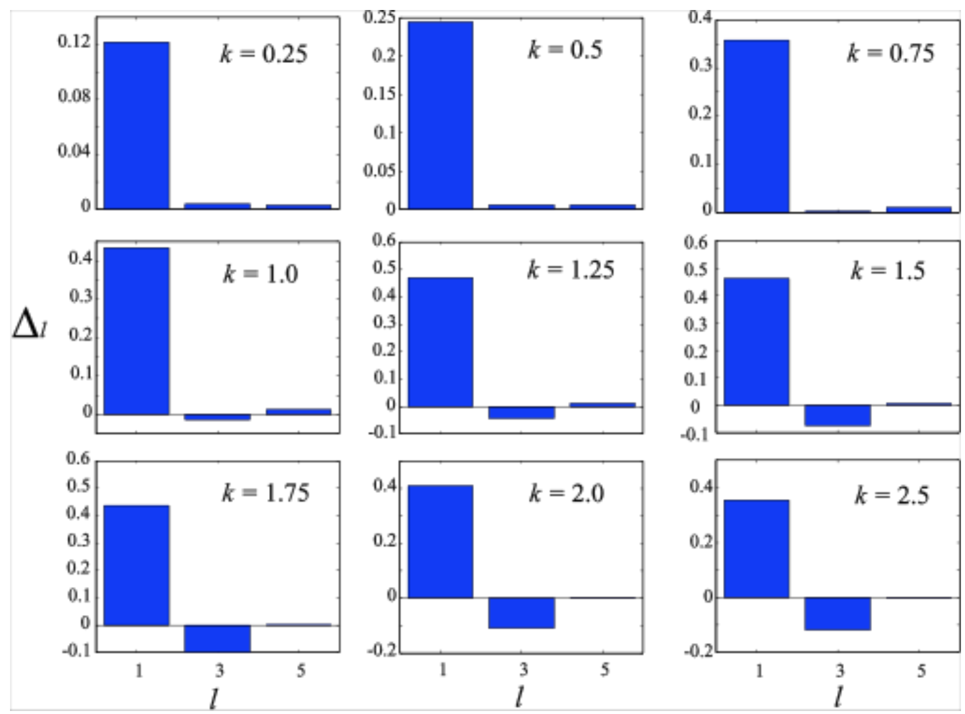


Angular distribution of order parameter

$$\Delta(\vec{k}) = \sum_{\text{odd } l} \Delta_l(k) Y_{l0}(\theta)$$

Order parameter





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  - Ideal gas: isotropic expansion
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Open questions

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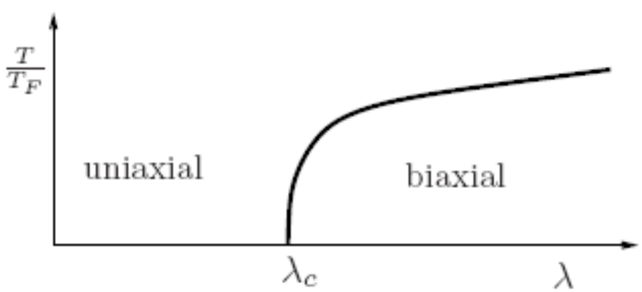

$$\lambda = nd^2 / E_F = \frac{2}{\left(6\pi^2\right)^{2/3}} C_{dd} \approx \frac{2}{15} C_{dd}$$

$$\lambda_c \approx 1$$

Fregoso et al., arXiv:0902.0739

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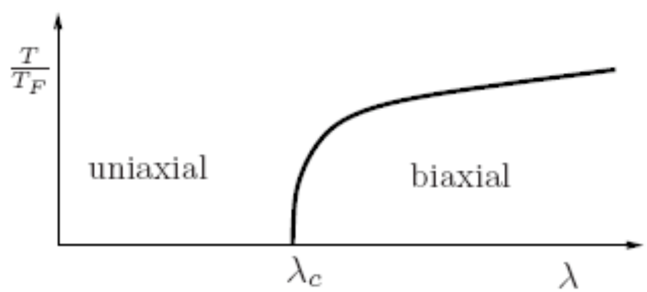

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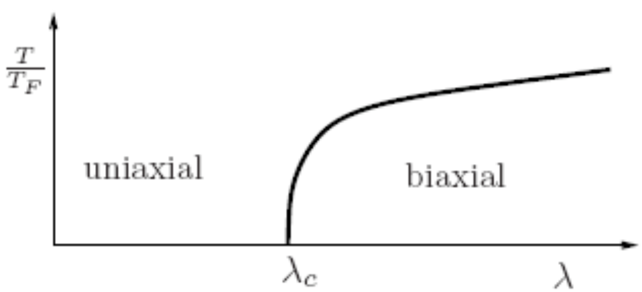

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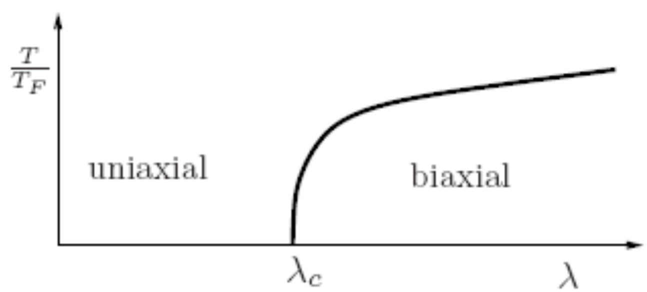

$$\lambda = nd^2 / E_F = \frac{2}{\left(6\pi^2\right)^{2/3}} C_{dd} \approx \frac{2}{15} C_{dd}$$

$$\lambda_c \approx 1$$

Fregoso et al., arXiv:0902.0739

Open questions

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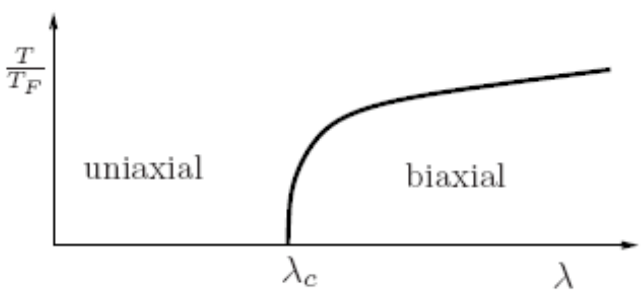

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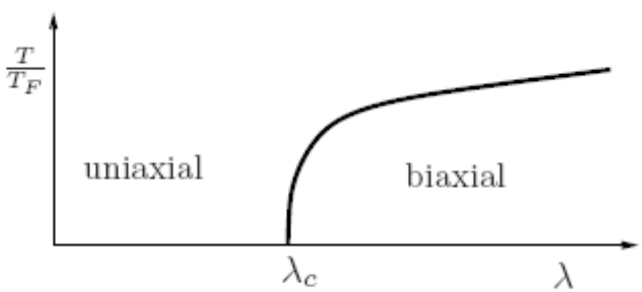

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