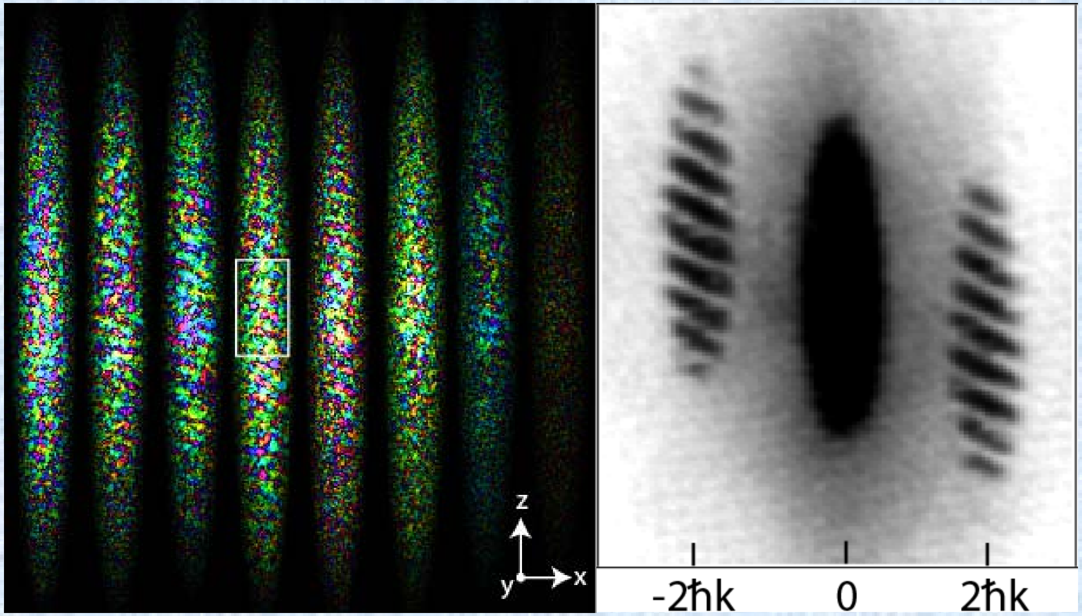
Spontaneous spatial order in a dipolar quantum fluid



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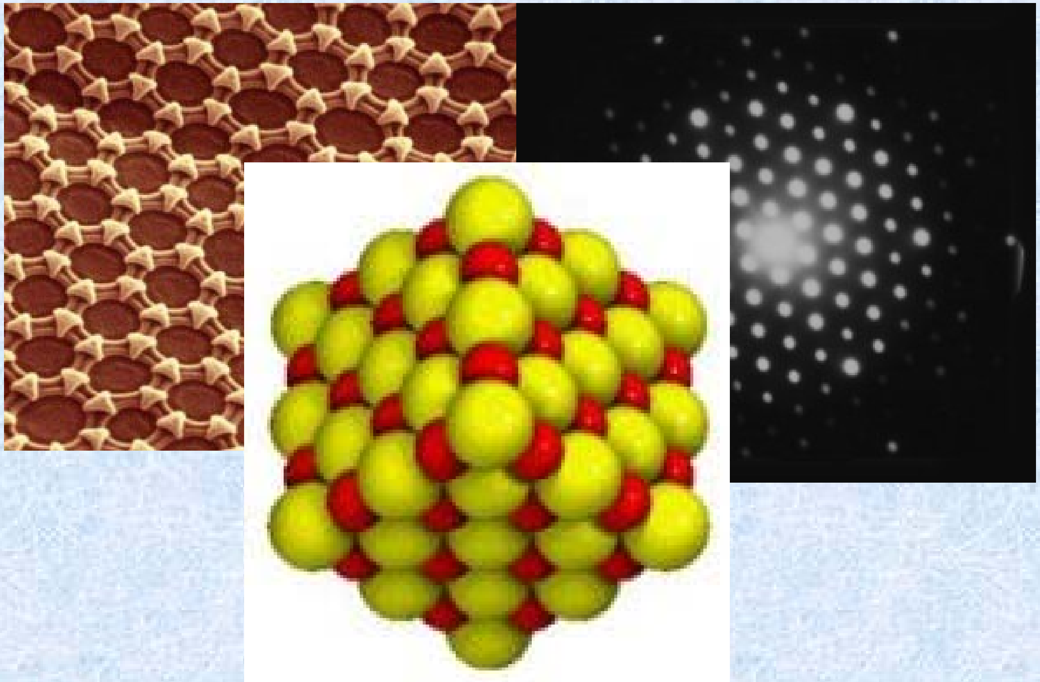
Jennie Guzman, Kater Murch, Dan Stamper-Kurn

Cornell University

Supersolids – spatially ordered superfluids

SOLIDS

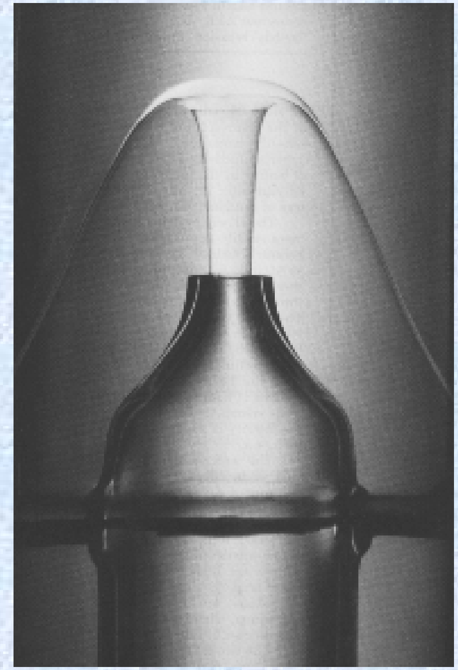


- Diagonal long range order
- Broken translational symmetry
- Order in real space

$$G(\vec{r}) = \left\langle \hat{\rho}_{\vec{G}}(\vec{r})\hat{\rho}^{*}_{\vec{G}}(0) \right\rangle$$

➢ Shear modulus

SUPERFLUIDS



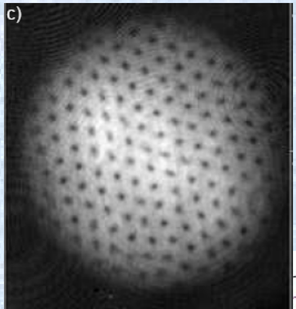
- Off-diagonal long range order
- Broken gauge symmetry
- Order in momentum space

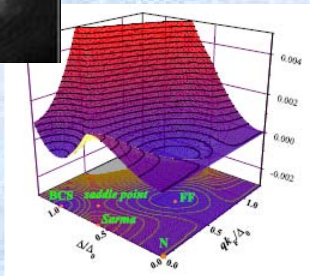
$$n(\vec{r}_1, \vec{r}_2) = \left\langle \hat{\Psi}^{*}(\vec{r}_1)\hat{\Psi}(\vec{r}_2) \right\rangle$$

➢ Frictionless flow

Quantum fluids with spontaneous spatial organization

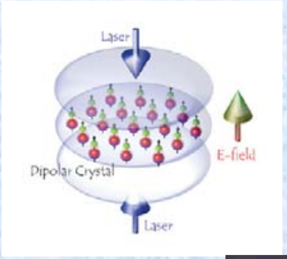
Rotating superfluids



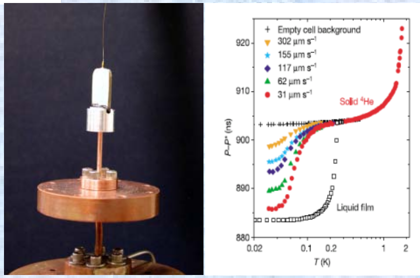


FFLO states

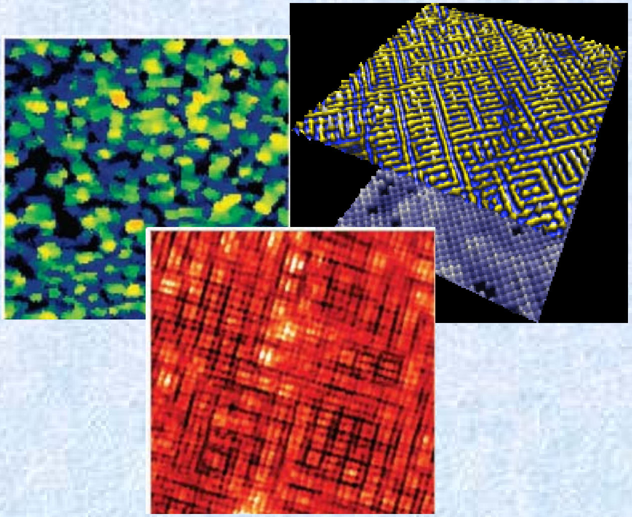
Dipolar molecules



'Supersolid' Helium



Cuprates, manganites



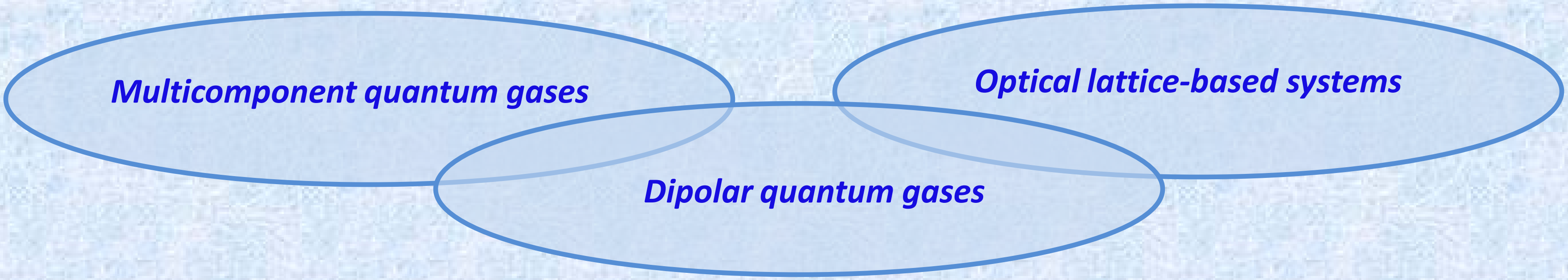
Increasing interaction strength

Ultracold realizations of spatially ordered quantum fluids

Is there a coherent quantum fluid that exhibits spontaneous spatial order in its ground state

AND

- *is weakly interacting*
- *has well known microscopic interactions*
- *is disorder-free*



Goral et al, PRL 88, 170406 (2002)
Sengupta et al, PRL 94, 207202 (2005)
Yi and Pu, PRL 97, 020401 (2006)
Menotti et al, PRL 98, 235301 (2007)

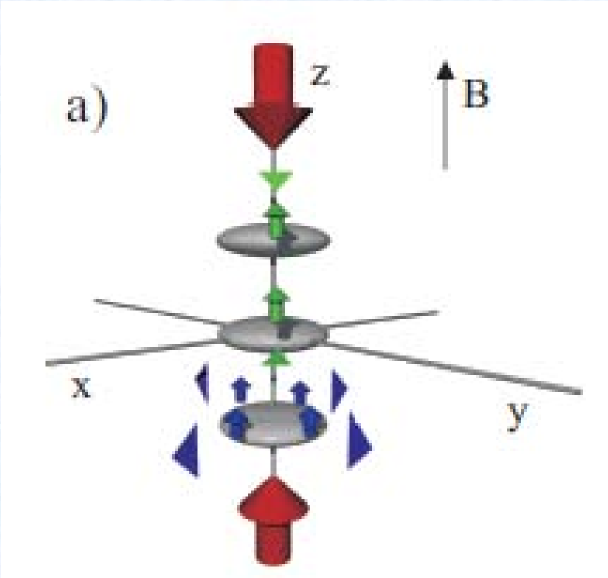
Dang et al, PRB 78, 132512 (2008)
Danshita et al, cond-mat/0804.0494 (2008)
Mathey et al, PRA 79, 011602 (2009)
… and many more.

Key ingredients

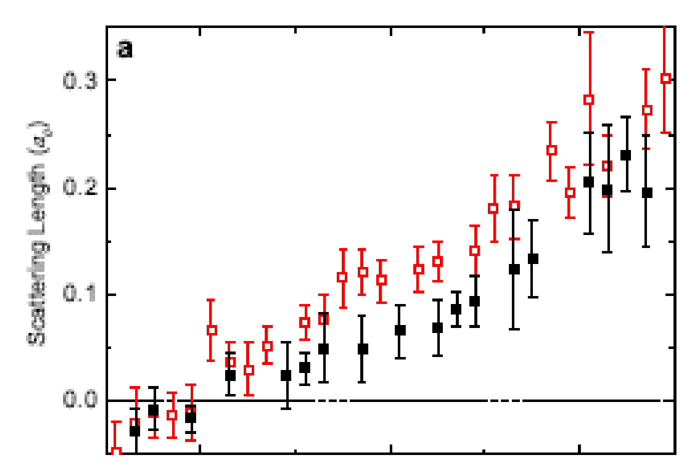
- ***Competing quantum phases***
- ***Long range interactions***
- ***Thermodynamic stability***

…Features present in dipolar spinor Bose gases.

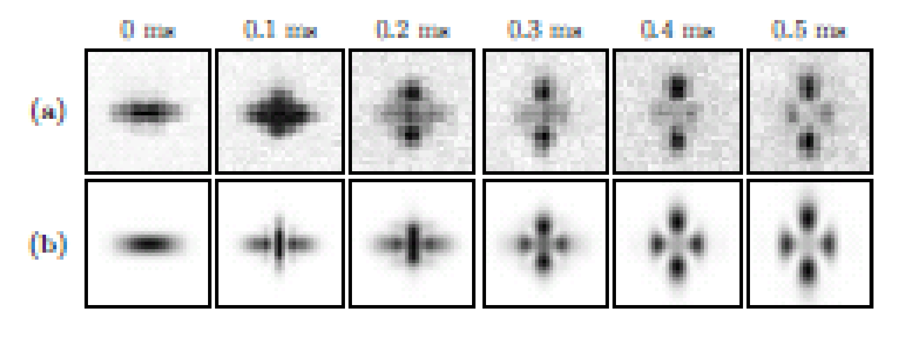
Dipolar quantum gases … a rapidly growing field



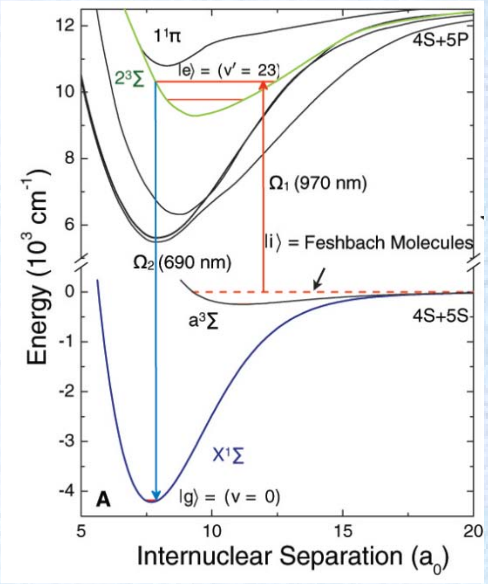
Dipole interactions in a K BEC, Fattori et al, Phys.Rev.Lett. 101, 190405 (2008)



Dipole interactions in a Li BEC, Pollack et al, Phys.Rev.Lett. 102, 090402,(2009)



d-wave collapse of a dipolar BEC, Lahaye et al Phys. Rev. Lett. 101, 080401 (2008)



Ultracold polar molecules, Ni et al, Science 322, 231 (2008)

Dipolar spinor Bose gases – a (very) brief review

Rotational invariance

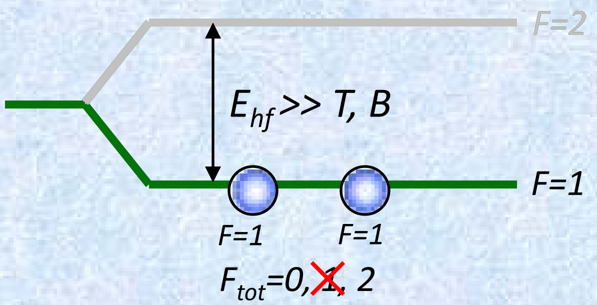
Bose statistics

→ *Effective low-energy contact interaction*

$$U(\vec{r}) = (g_0 + g_2 \vec{F}_1 \bullet \vec{F}_2) n(\vec{r}) \delta(\vec{r})$$

$$\begin{cases} g_0 = \dfrac{4\pi\hbar^2}{m}\dfrac{2a_2 + a_0}{3}, \; g_2 = \dfrac{4\pi\hbar^2}{m}\dfrac{a_2 - a_0}{3} \\ g_2 << g_0 \end{cases}$$

F=1 spinor gases



*T. L. Ho, PRL **81**, 742 (98),*
*T. Ohmi and K. Machida, J. Phys. Soc. Jpn. **67**, 1822, (98)*

$g_2 > 0$ □ *'Polar' spinor eg. F=1 ^{23}Na*
$g_2 < 0$ □ *Ferromagnetic spinor eg. F=1 ^{87}Rb*

N. N. Klausen, J. H. Bohn, C. H. Greene, PRA 64, 053602 (01)
J. Stenger et al, Nature 396, 345 (98)
H. Schmaljohann et al, Phys. Rev. Lett. 92, 040402 (04)
M.-S. Chang et al, Nature Phys. 1, 111 (05)

Are magnetic dipole interactions relevant?

Energy scale $\varepsilon_d \approx \mu_0 \mu_B^2 n_{3d} \sim k_B 100$ *pK (much smaller than the chemical potential)*

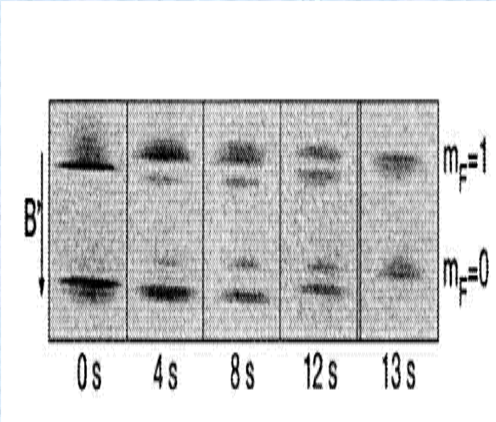
But, in a Rubidium spinor gas : $\varepsilon_d \sim g_2 n$

So, in a spinor fluid, dipolar interactions should break the rotational symmetry leading to new ground states.

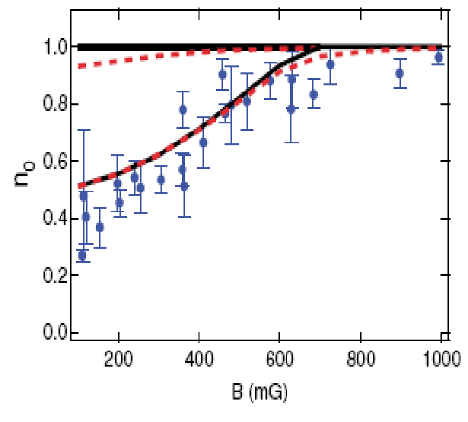
S. Yi, L. You, H. Pu, Phys. Rev. Lett. 93, 040403 (04)
Y. Kawaguchi et al, Phys. Rev. Lett. 98, 110406 (07)

Magnetic phases of a spinor gas

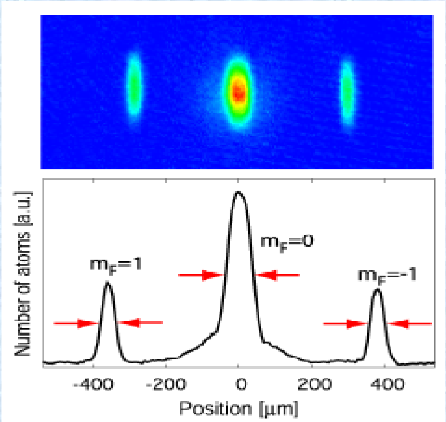
- *Anti-ferromagnetic (^{23}Na) and Ferromagnetic (^{87}Rb) phases observed.*
- *Coherent spin exchange collisions.*



*PRL **82**, 2228 (1999)*
Ketterle Group



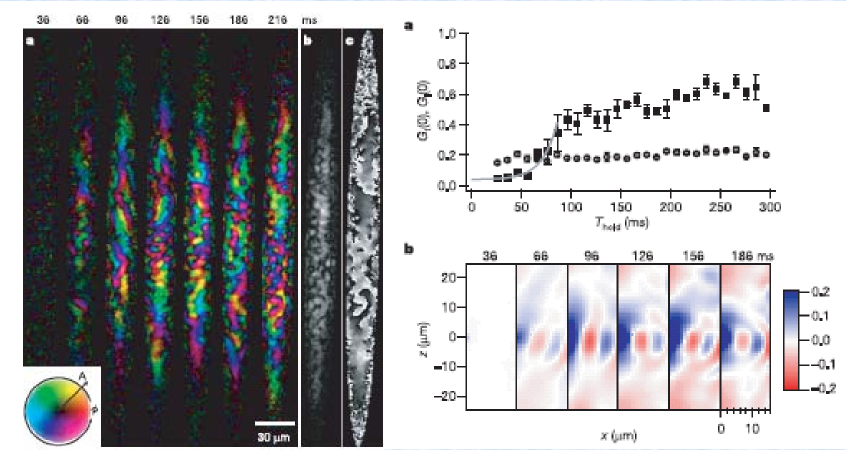
*PRL **92**, 140403 (2004)*
Chapman Group



Appl. Phys. B 79, 1001 (05)
Sengstock Group

- *Dynamics past a quantum phase transition studied by magnetization-sensitive imaging.*

*L. E. Sadler et al, Nature **443**, 312 (2006)*



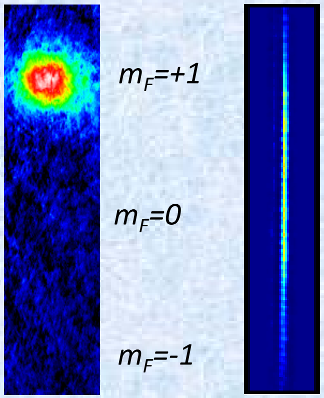
Experiments seem to indicate spinor behavior dictated solely by contact interactions.

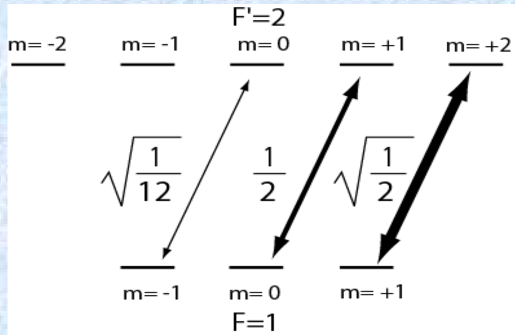
... Need sensitive probe of `lower energy’ physics.

Detection of a magnetic quantum fluid by phase contrast imaging

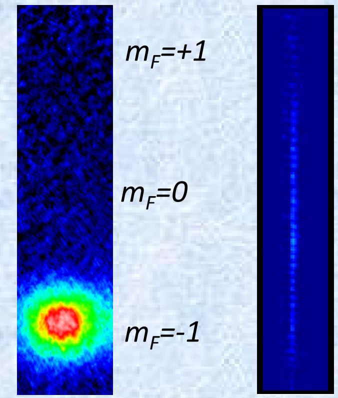
Longitudinal magnetization

$$\Psi = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

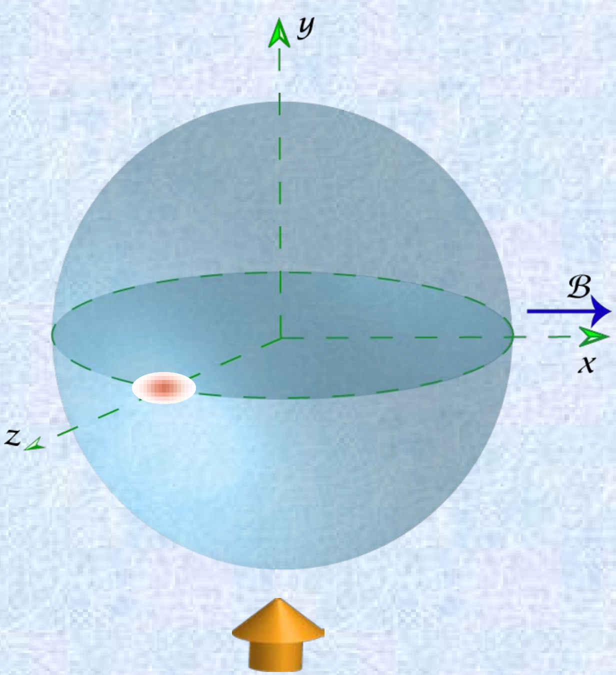




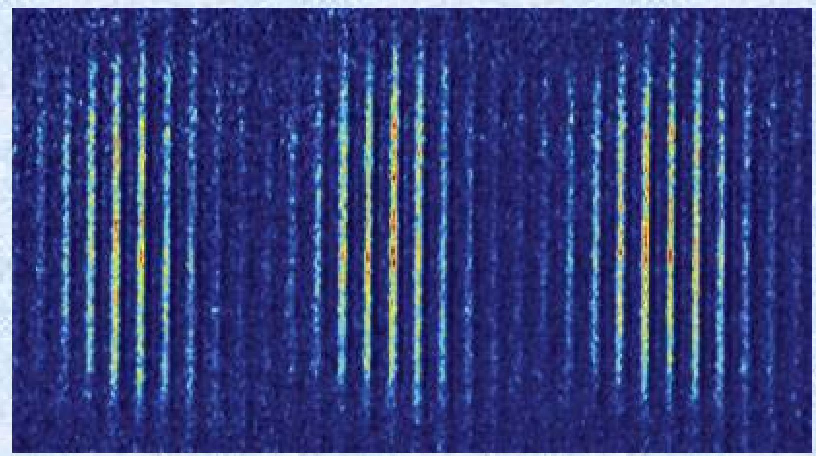
$$\Psi = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



Imaging the transverse magnetization using circular birefringence



$$\Psi = \begin{pmatrix} \frac{1}{2} e^{-i\omega_L t} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{2} e^{+i\omega_L t} \end{pmatrix}$$

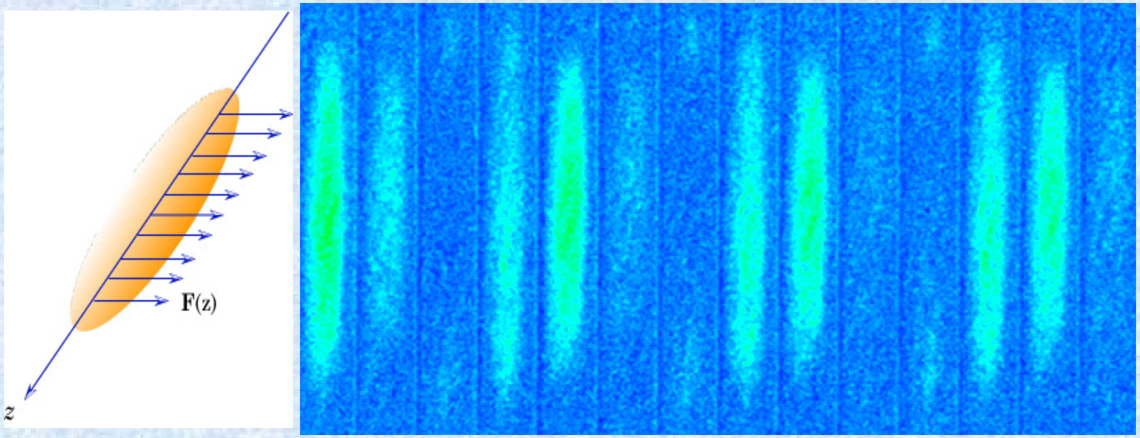


Spatially resolved Larmor precession – $F_t(x,z)$

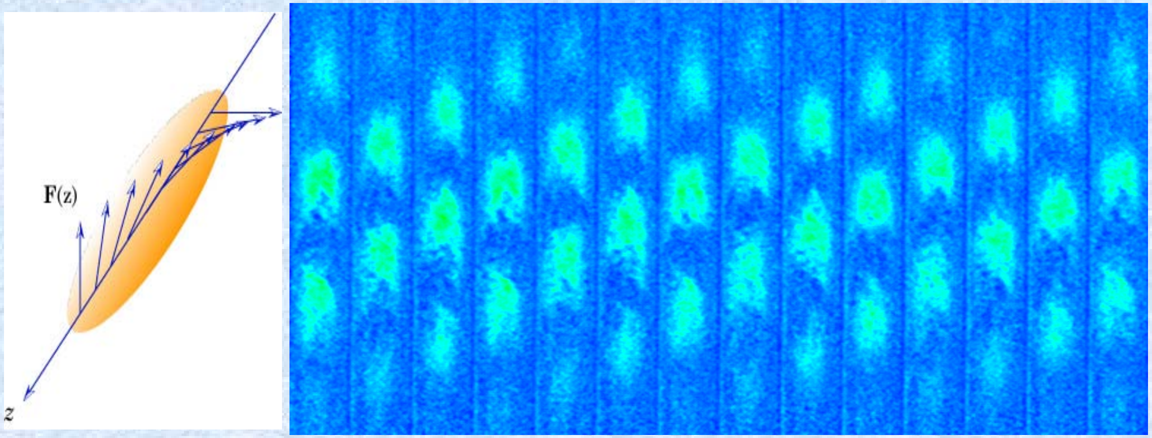
*E. Mueller and I. Carusotto, J. Phy. B **37**, S115 (2004).*
*J. M. Higbie et al, PRL **95**, 050401 (2005)*
*M. Vengalattore et al, PRL **98**, 200801 (2007)*

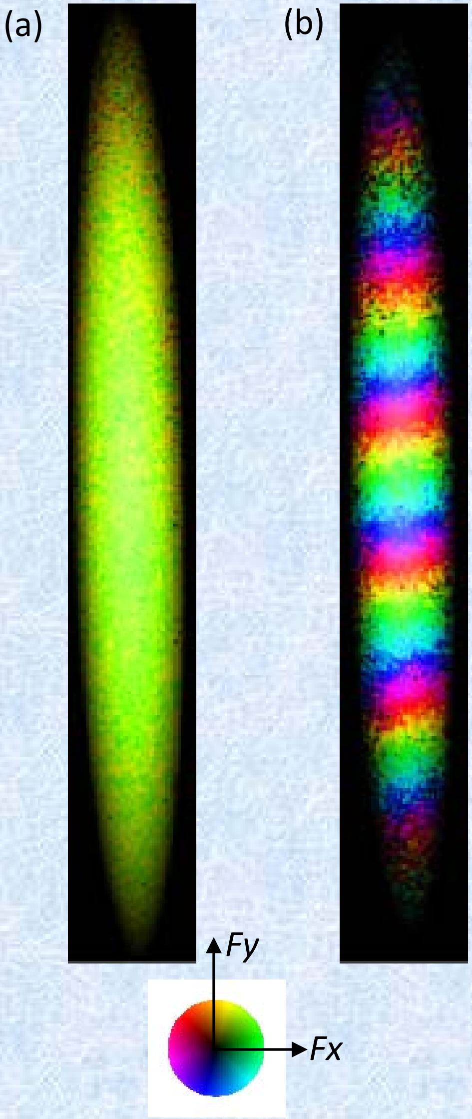
Extracting the magnetization from nondestructive images

(a) *Homogeneous magnetization*

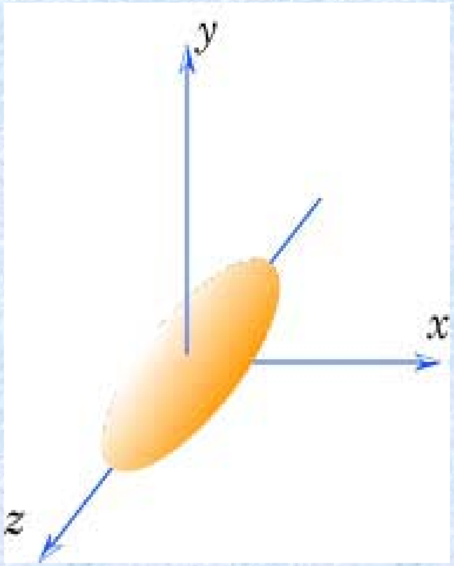


(b) *Helical magnetization*





Experimental parameters



$N = 3 \times 10^6$
$(\omega_x, \omega_y, \omega_z) = 2\pi\ (56, 356, 4.5)\ s^{-1}$
$n_0 = 2 \times 10^{14}\ cm^{-3}$

Spin mixing energy $\varepsilon = g_2 n \sim k_B \times 400$ pK

Length scale:

Spin healing length

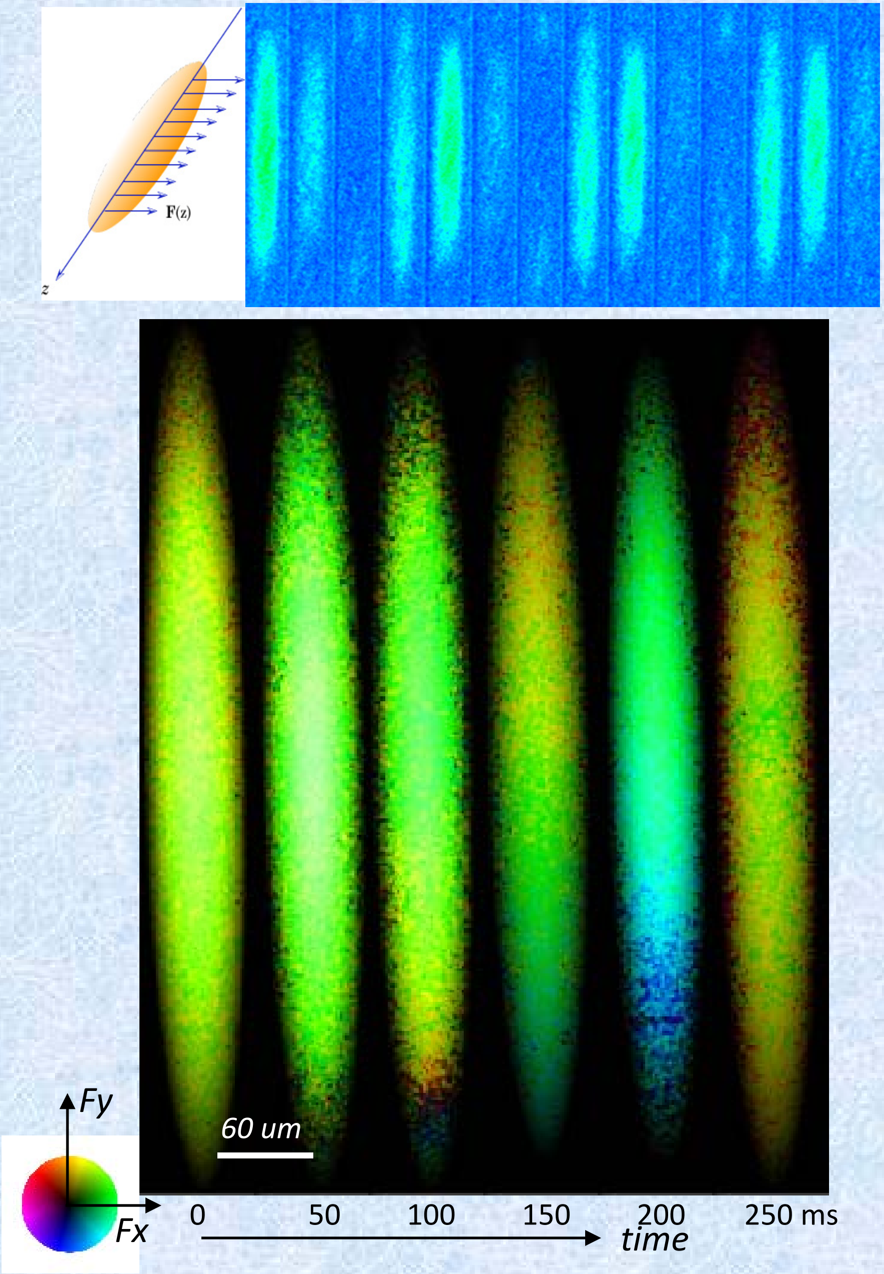
$$\xi_s = \sqrt{\frac{\hbar^2}{2m\varepsilon}} \sim 2.6\,\mu\text{m}$$

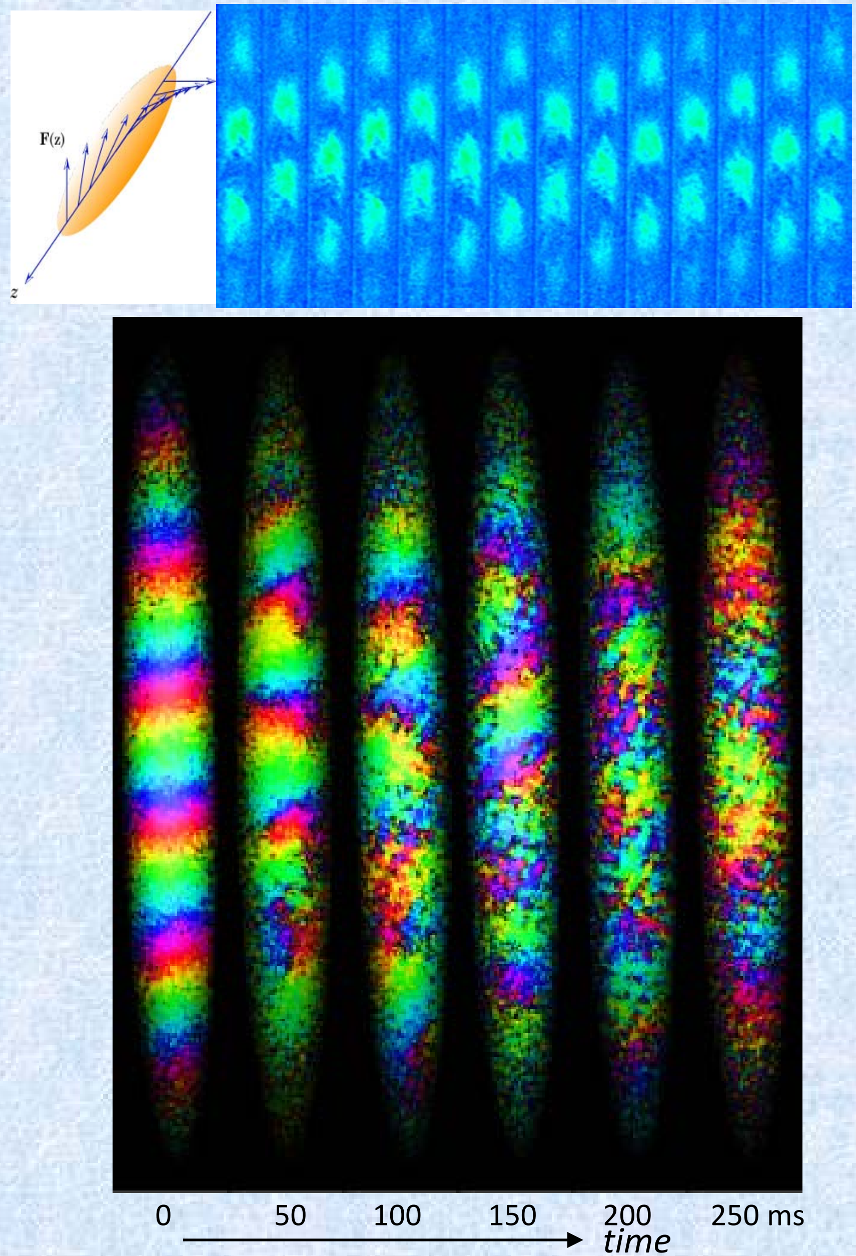
Time scale:

$$\tau_s = \frac{h}{\varepsilon} \sim 13 \text{ ms}$$

Thomas-Fermi radii = $(r_x, r_y, r_z) \sim (6, 0.9, 75)\ \xi_s$
Anisotropic trap results in 2D spin dynamics

Dynamical instabilities toward a spatially modulated phase

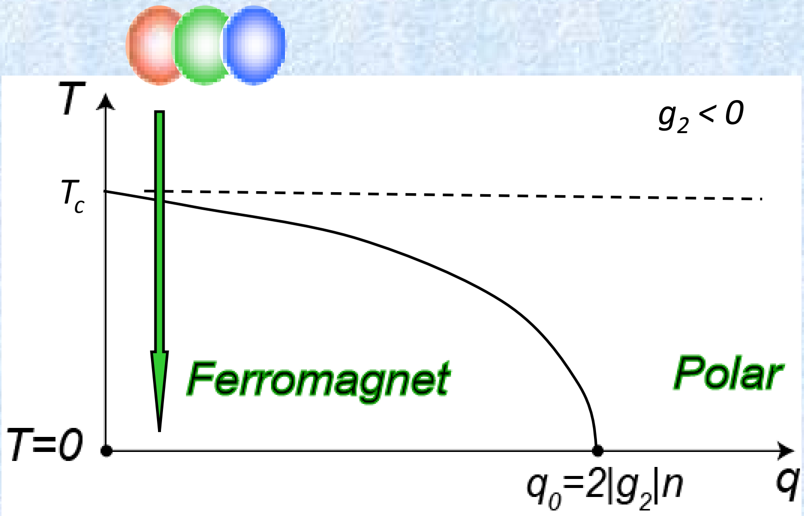


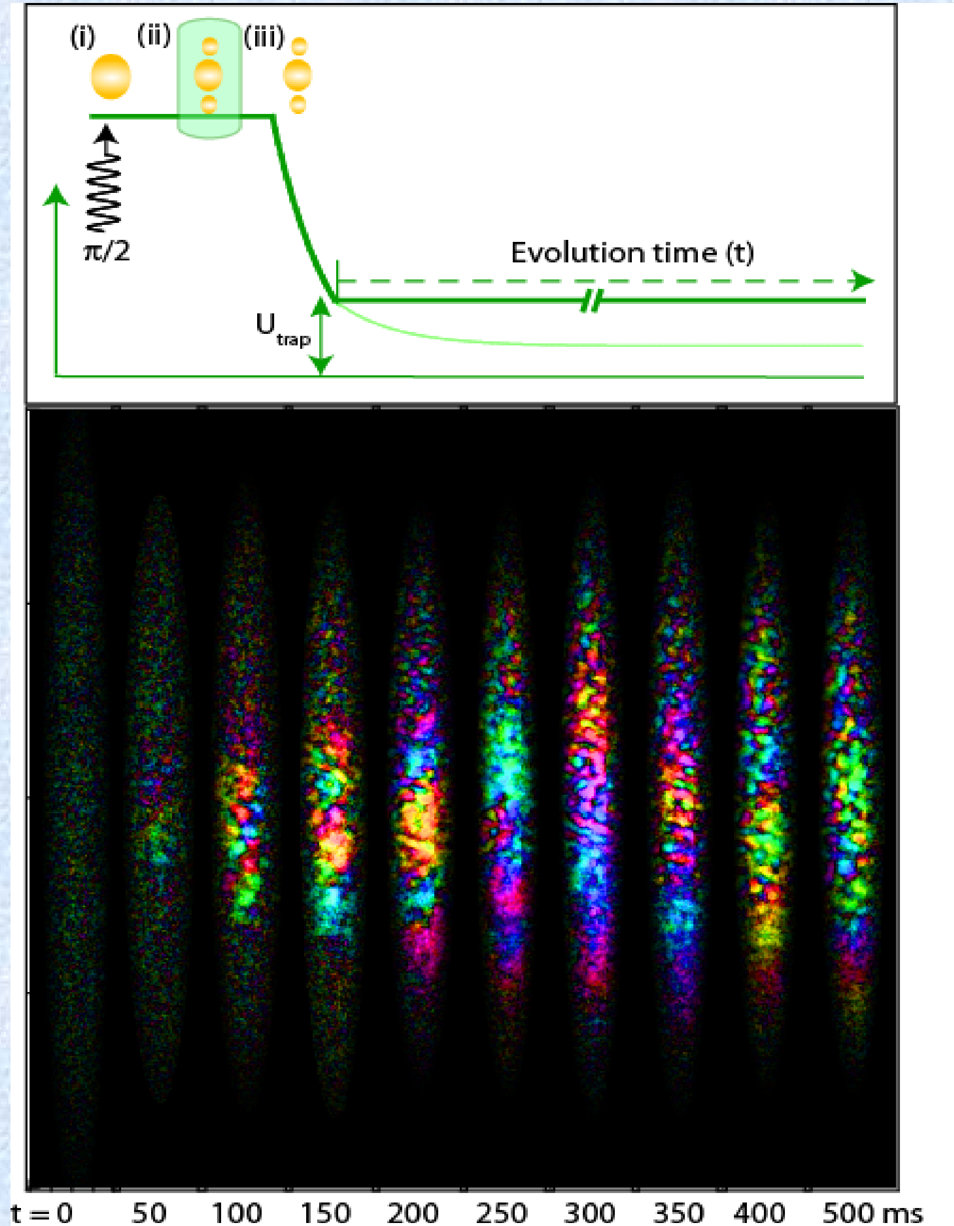


M. Vengalattore et al, PRL 100, 170403 (2008)

Is this spin texture the 'true ground state' of the dipolar spinor?

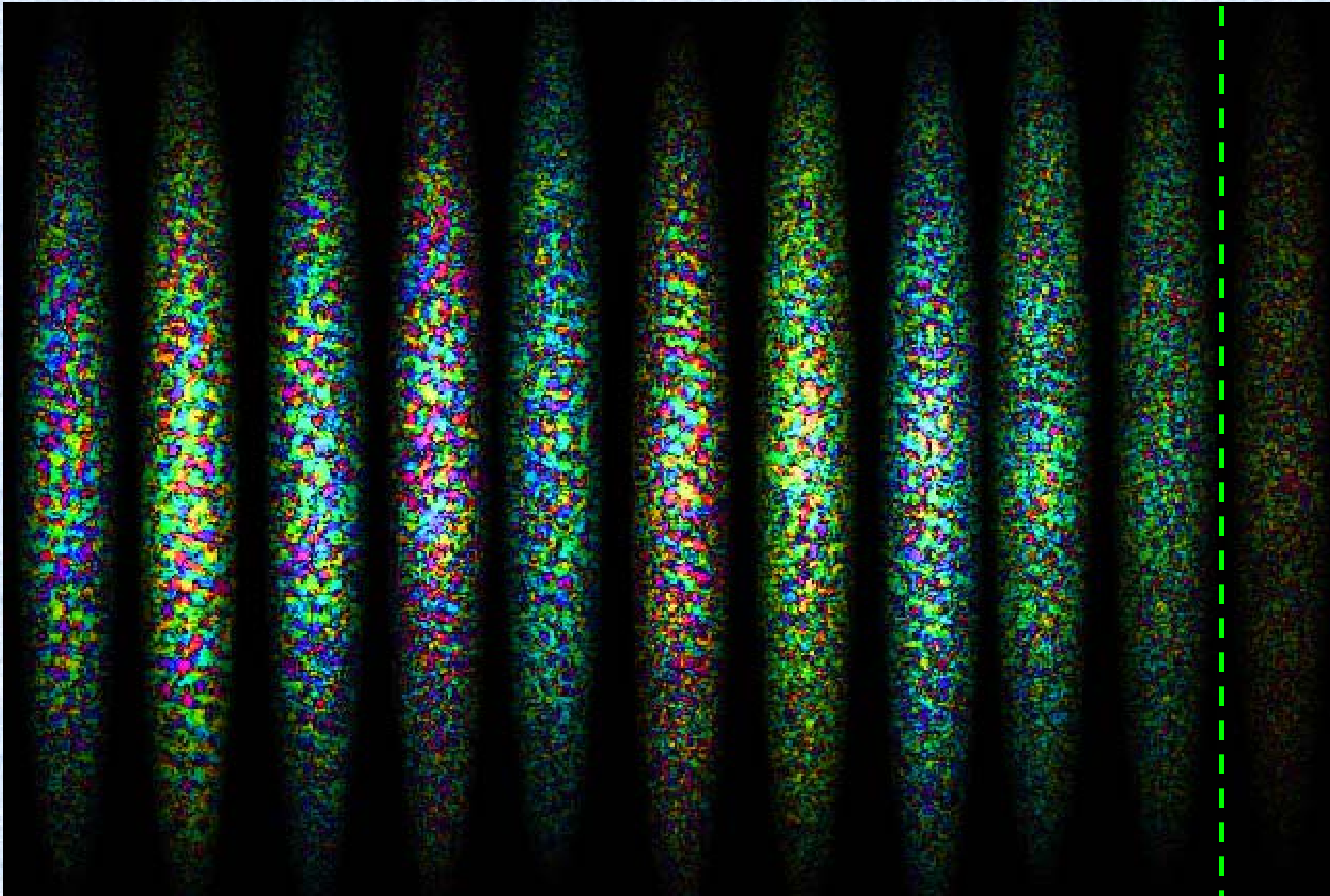
Cool an incoherent thermal mixture below the transition temperature



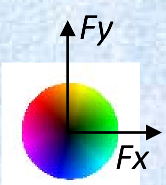


What do we expect?
If we neglect long-range interactions, the ground state should be a condensate that is
(i) Transversely magnetized
(minimizes contact interactions)
(ii) Uniform
(minimizes kinetic energy)

Magnetic order for the gradually cooled thermal mixtures



T=Tc
(BEC)

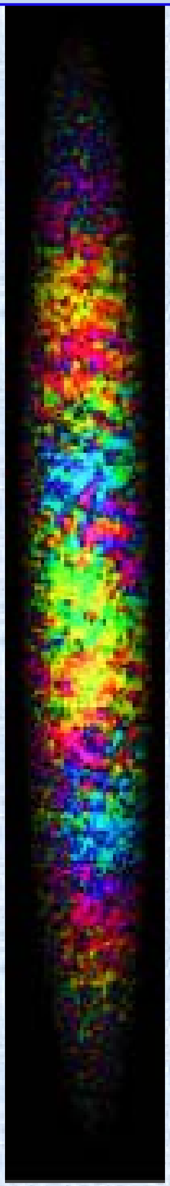


Increasing temperature

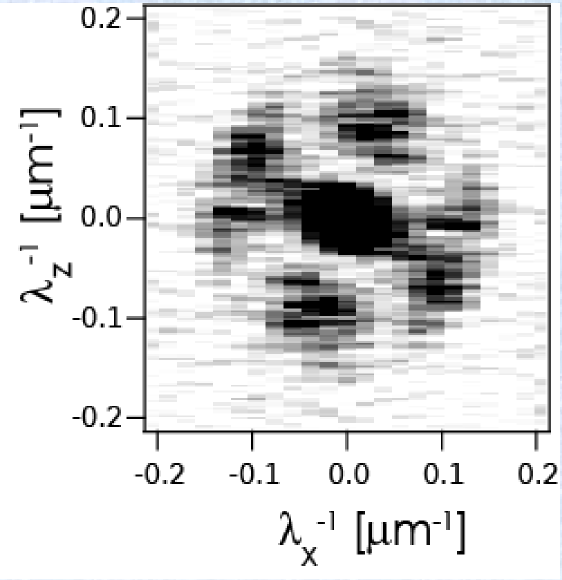
M. Vengalattore et al, arXiv:0901.3800 (2009)

A closer look at the spin textures

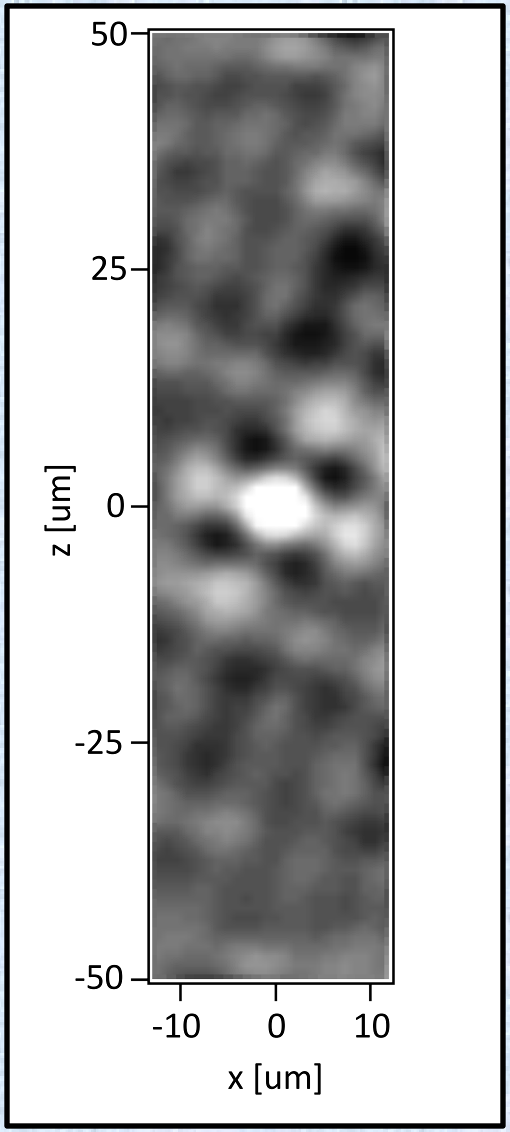
Magnetization density of the spinor gas



Spatial Fourier transform of the magnetization



Two-point spin correlation function



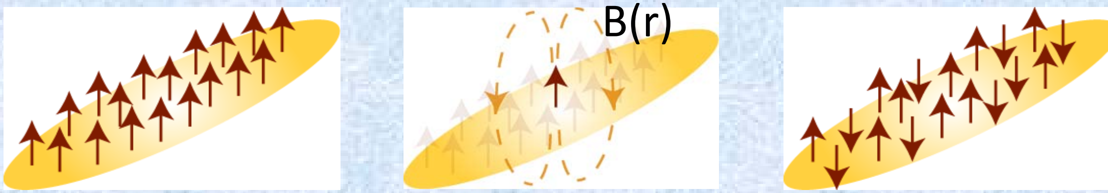
Spontaneous emergence of a magnetically ordered checkerboard phase.

Magnetic dipolar energy

Energy scale :

$$\varepsilon_{dip} = \mu_0 g_F^2 \mu_B^2 n \quad \sim 500 \ \ \text{pK} \sim |g_2| n$$

Would this explain the spontaneous modulation?



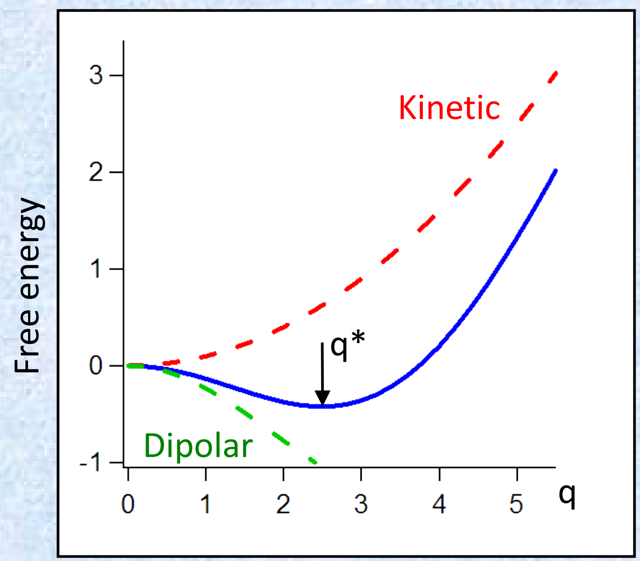
Aligned spins in a plane

Landau free energy for an Ising-like 2D system

$$F[\phi] = \int d^2x \left[\frac{\hbar^2}{2m} n_{2D} |\nabla \phi|^2 \right] + \mu_0 \mu_B^2 n_{2D}^2 \iint d^2x_1 d^2x_2 \frac{\phi(x_1)\phi(x_2)}{|x_1 - x_2|^3}$$

$$= \int d^2q \underbrace{\left[\frac{\hbar^2}{2m} n_{2D} q^2 + \mu_0 \mu_B^2 n_{2D}^2 G_q \right]} \phi_q \phi_{-q}$$

•This free energy is minimized at a non-zero wavevector q*
•This indicates that the homogenous ferromagnet is only metastable, and the gas should exhibit spontaneous modulations of the spin texture.

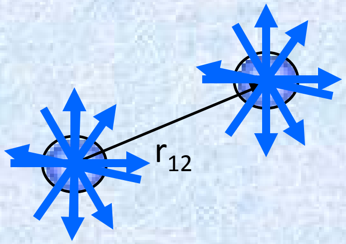


Are these textures induced by the magnetic dipolar interaction?

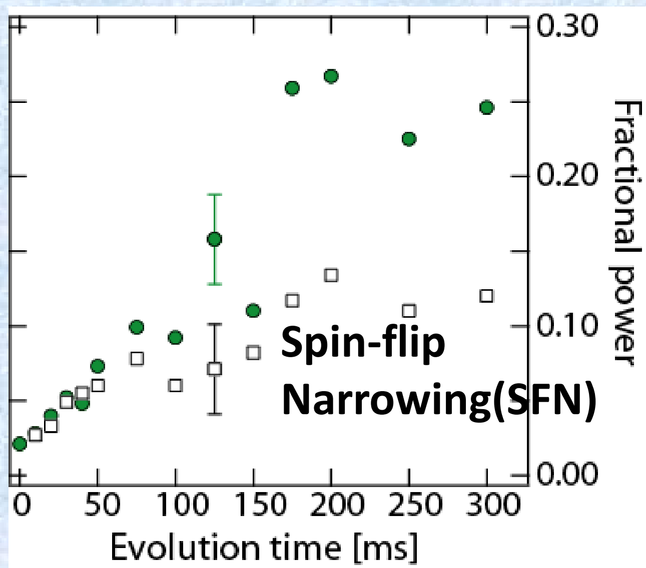
Dipolar interaction

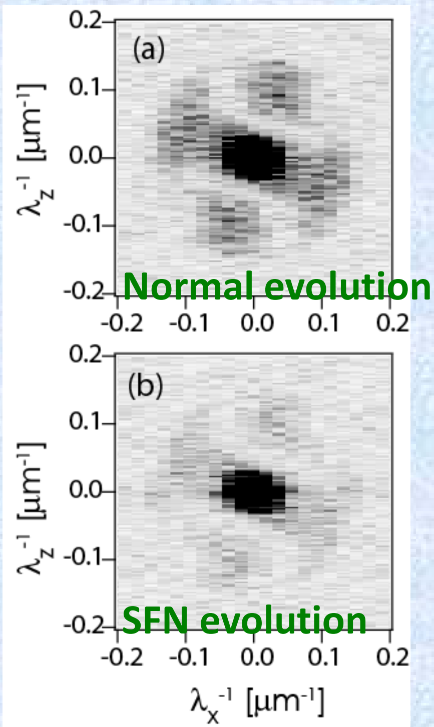
$$U_{dip}(\vec{r}_{12}) \propto \frac{\mu_0}{4\pi}(g\mu_B)^2 \frac{2\hat{F}_{1z}.\hat{F}_{2z} - \hat{F}_{1x}.\hat{F}_{2x} - \hat{F}_{1y}.\hat{F}_{2y}}{r_{12}^3}$$

•Impose common rotations on both dipoles about random axes. (Analogous to NMR technique of spin flip narrowing)



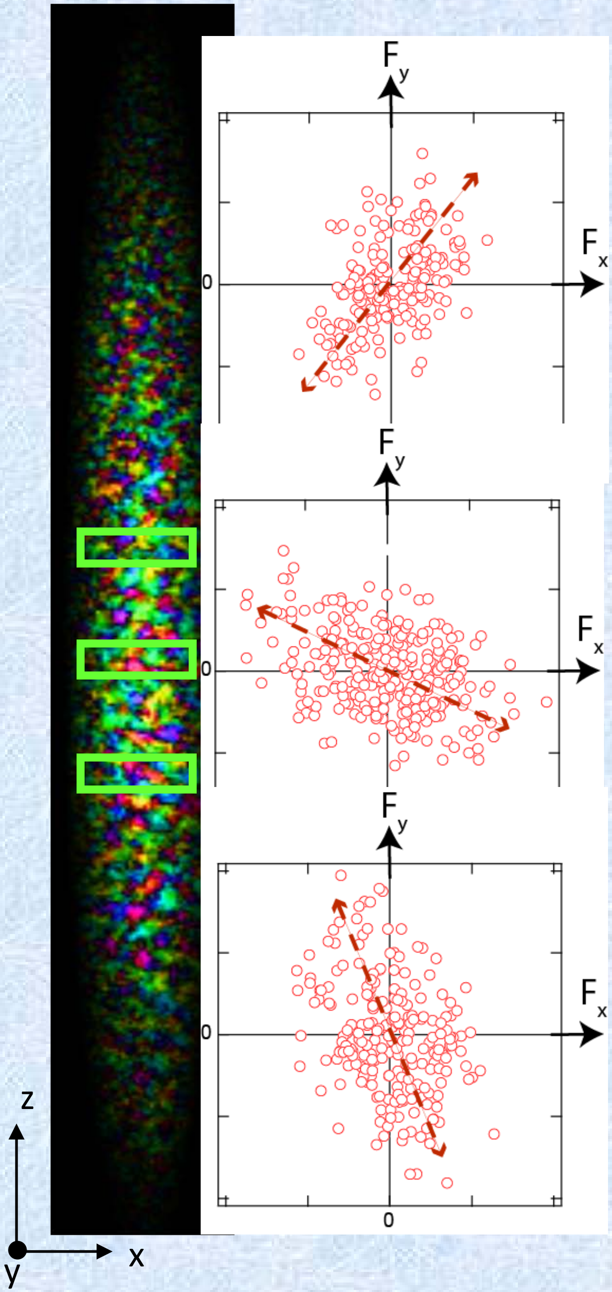
For stationary dipoles, the time-averaged pairwise interaction vanishes.



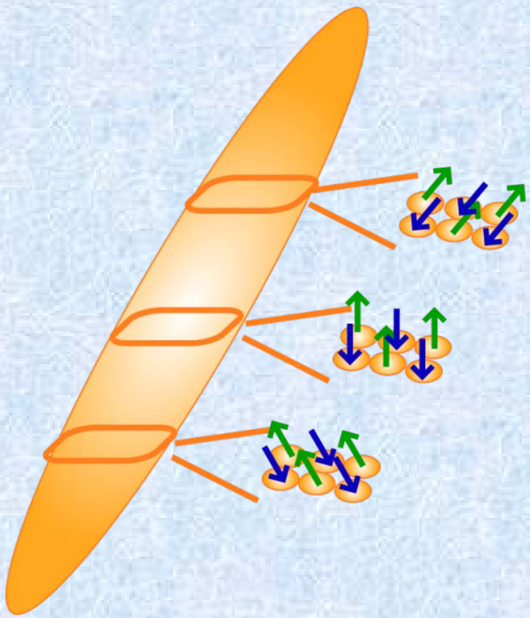


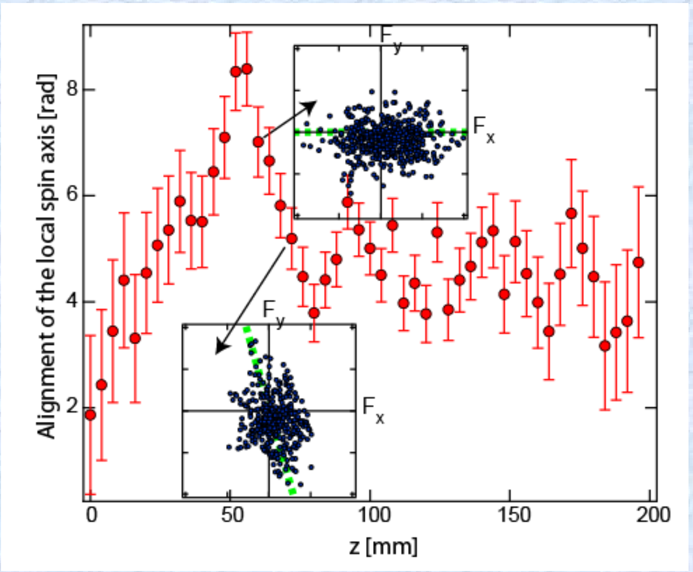
The spontaneous modulation is suppressed when the dipolar interaction is reduced.

An emergent spin axis in the crystal



The spin lattice seems to be locally ‘Ising-like’.
The local orientation of this Ising lattice shows a gradual spatial variation.

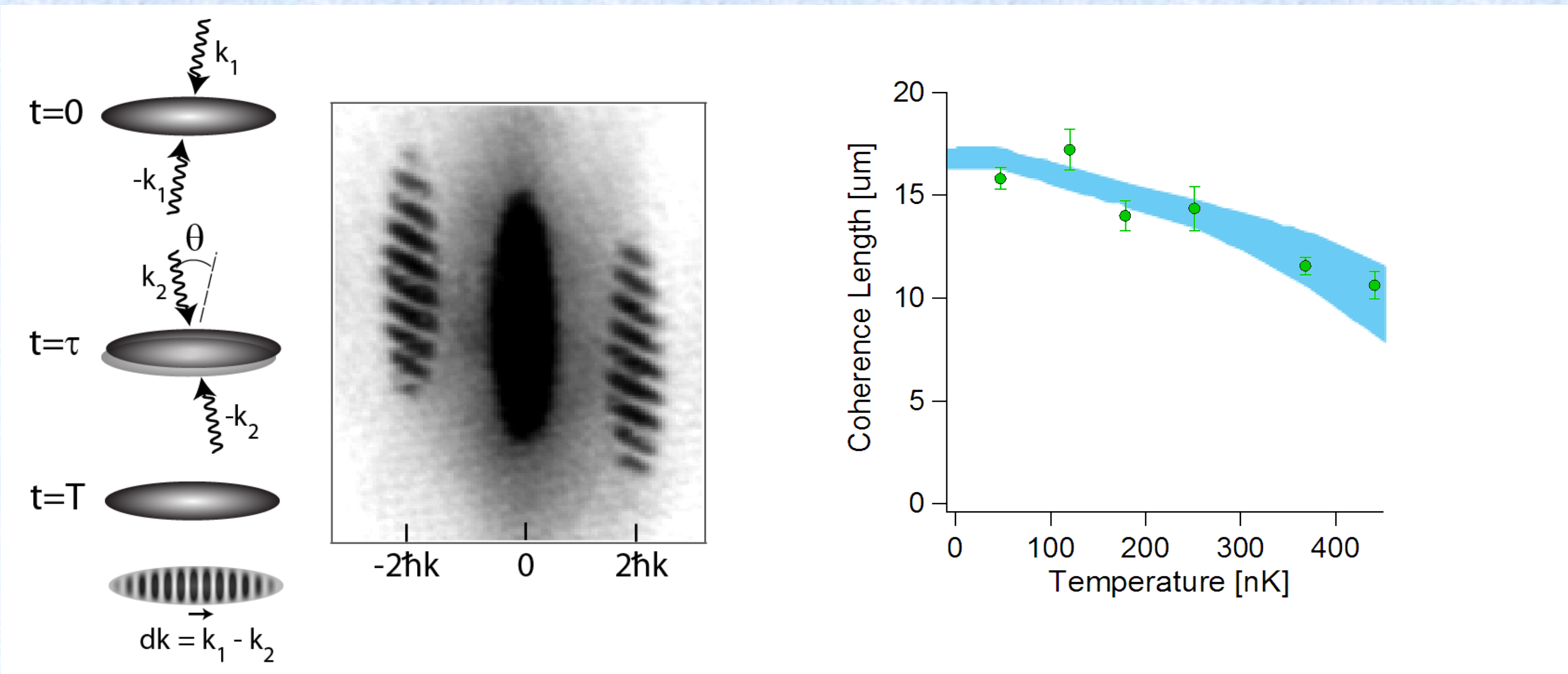




Does the magnetic crystal exhibit long range phase coherence?

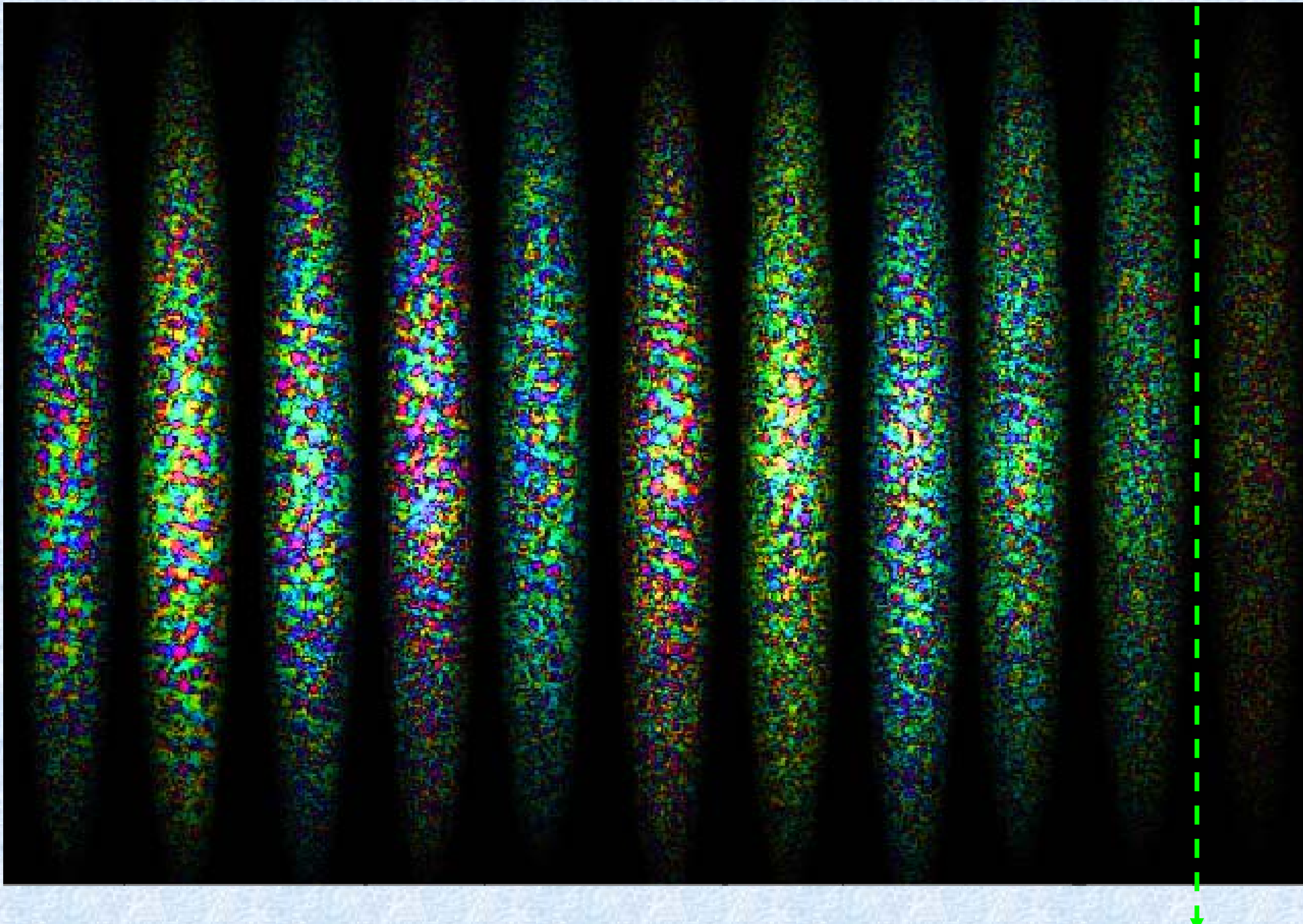
- *Atom interferometry using Bragg pulses.*
- *A Ramsey sequence with a tunable time delay is a direct measure of first-order coherence.*

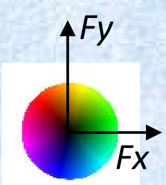
➢ *Use 2-wavevector sequence to realize a heterodyne probe.*
(ideas similar to W. Phillips's group, P. Clade' et al, arXiv:0805.3519)



Coherence length is consistent with the spatial extent of the spinor gas.

Magnetic order for (1/3,1/3,1/3) mixtures

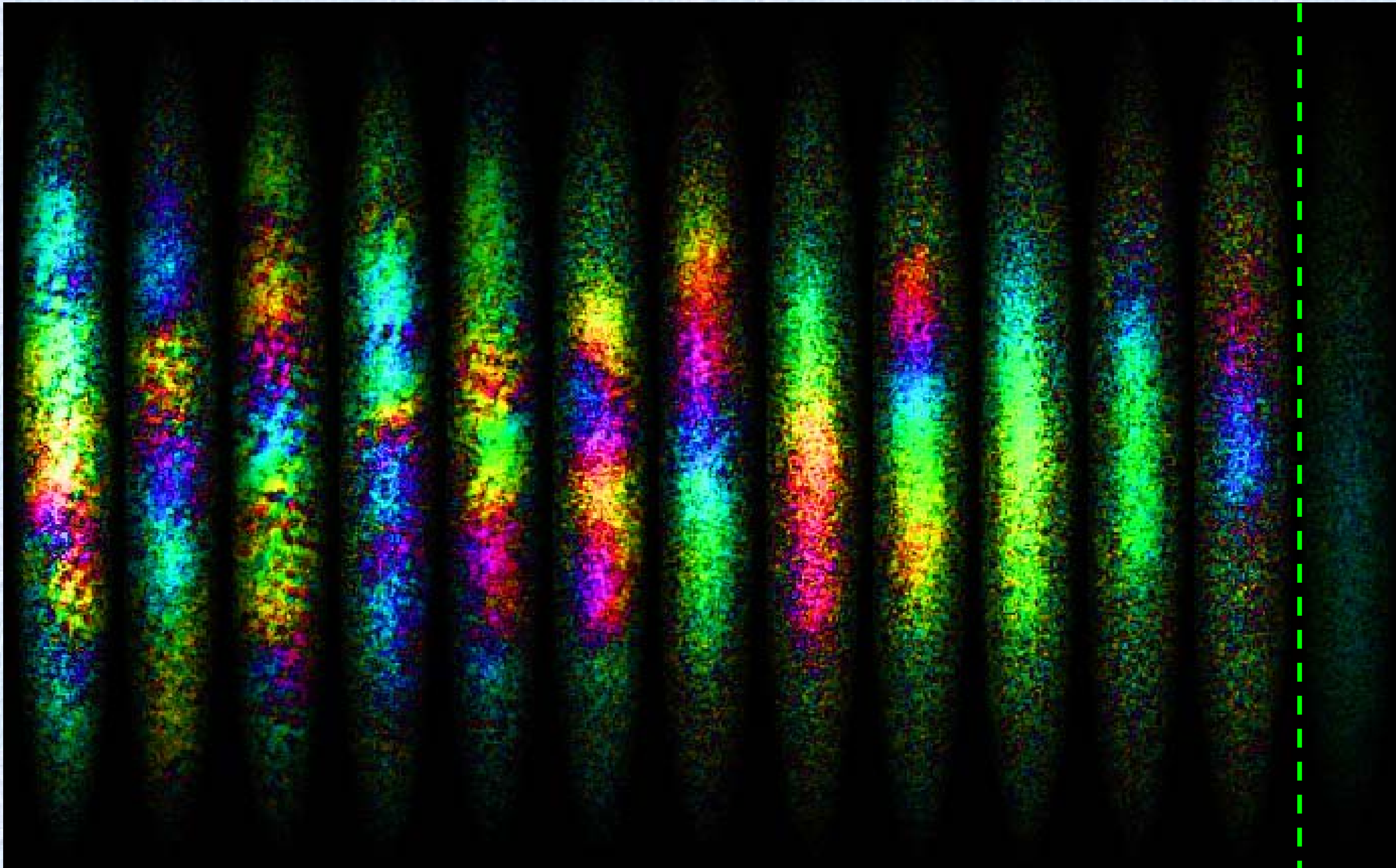


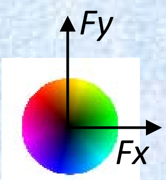


Increasing temperature

T=Tc
(BEC)

Magnetic order for (1/4,1/2,1/4) thermal mixtures

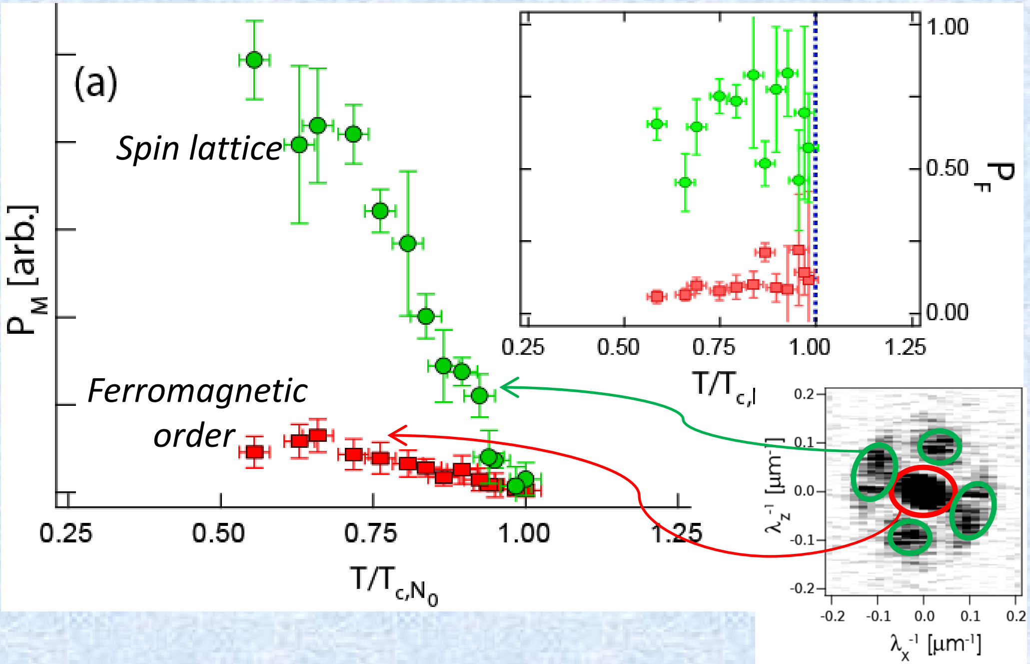




Increasing temperature

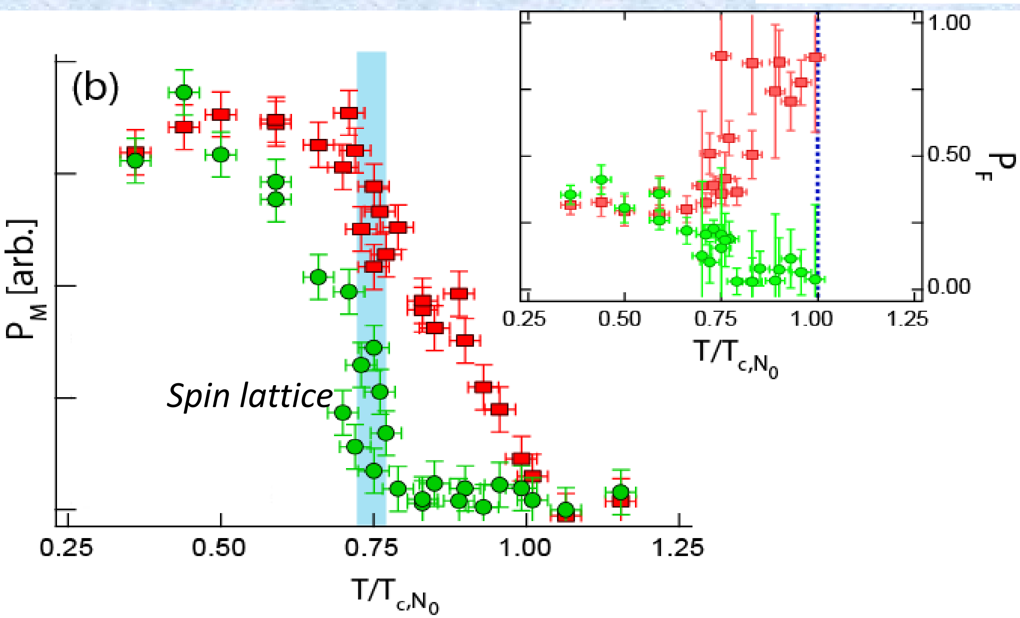
T=Tc
(BEC)

Magnetic order parameters at finite temperature



For initial $\rho = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{pmatrix}$

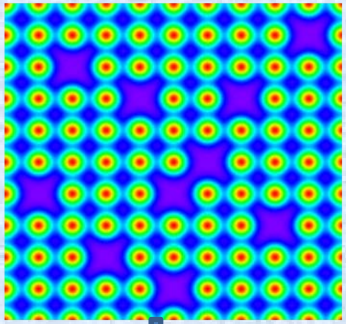
- See self-organized textures immediately below Tc.
- Homogeneous (or long range) ferromagnetic order seems to be negligibly small at all temperatures.
- The magnetization samples all of spin-space uniformly.



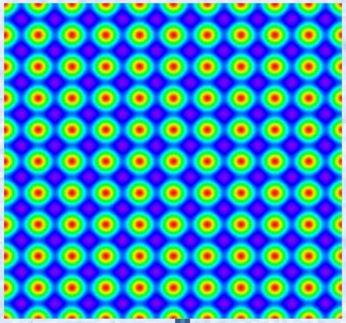
For initial $\rho = \begin{pmatrix} 1/4 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{pmatrix}$

- Only ferromagnetic order evident immediately below Tc.
- Self-organized texture appears below a second transition.
- The magnetization is predominantly in the transverse plane.

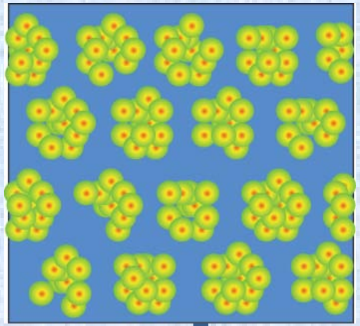
Do we fully understand this system?



Bose condensate of defects,
$N_{atoms} < N_{sites}$



Commensurate solid, (No superfluidity)
$N_{atoms} < N_{sites}$



"Coherent" solid
$N_{atoms} >> N_{sites}$

Long wavelength spin dynamics of ferromagnetic condensates

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(Dated: August 27, 2008)

We obtain the equations of motion for a ferromagnetic Bose condensate of arbitrary spin in the long wavelength limit. We find that the magnetization of the condensate is described by a non-trivial modification of the Landau-Lifshitz equation, in which the magnetization is advected by the superfluid velocity. This hydrodynamic description, valid when the condensate wavefunction varies on scales much longer than either the density or spin healing lengths, is physically more transparent than the corresponding time-dependent Gross-Pitaevskii equation. We discuss the conservation laws of the theory and its application to the analysis of the stability of magnetic helices and Larmor precession. Precessional instabilities in particular provide a novel physical signature of dipolar forces. Finally, we discuss the anisotropic spin wave instability observed in the recent experiment of Vengalattore *et. al.* (Phys. Rev. Lett. **100**, 170403, (2008)).

Roton softening and supersolidity in Rb spinor condensates

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Superfluids with a tendency towards periodic crystalline order have both a phonon and roton like spectrum of collective modes. The softening of the roton spectrum provides one route to a supersolid. We show that roton softening occurs in ^{87}Rb spinor condensates once dipolar interactions and spin dynamics are taken into account. By including the effects of a quasi-two-dimensional geometry and rapid Larmor precession, we show a dynamical instability develops in the collective mode spectrum at finite wavevectors. We construct phase diagrams showing a variety of instabilities as a function of the direction of the magnetic field and strength of the quadratic Zeeman shift. Our results provide a possible explanation of current experiments in the Berkeley group *Phys. Rev. Lett.* 100:170403 (2008).

Spontaneous Vortex Lattices in Quasi 2D Dipolar Spinor Condensates

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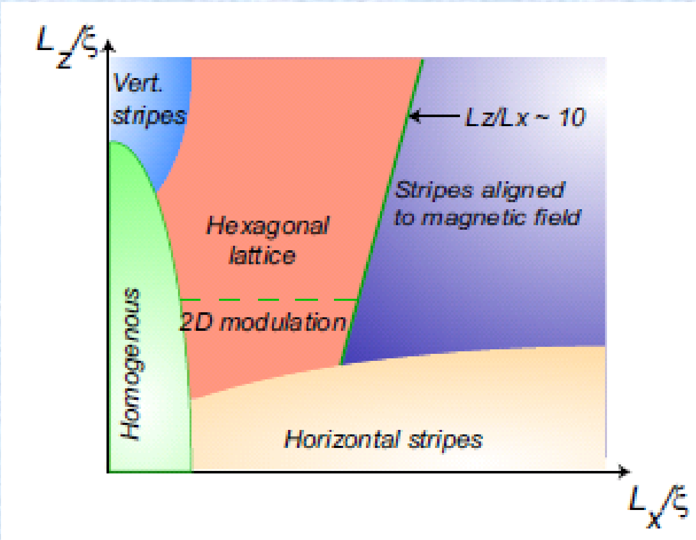
The ground state of the dipolar spinor gas

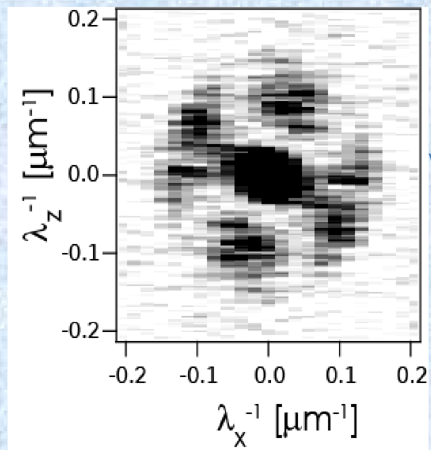


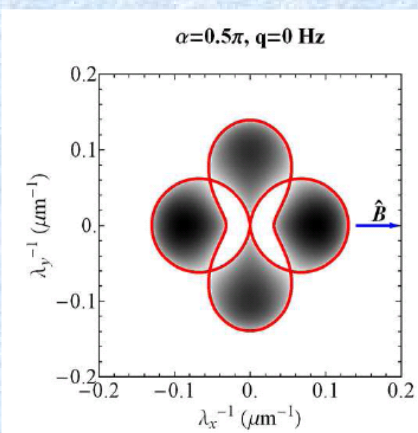
In the limit of predominantly transverse magnetization, the Dipolar interaction can be written in the form

$$\langle U_d(r_{12})\rangle = \frac{c_d}{r_{12}^3}\hat{F}_{\perp,1}\hat{F}_{\perp,2}\cos(\theta_1-\theta_2)\left(1-\frac{3}{2}e_x^2\right)$$

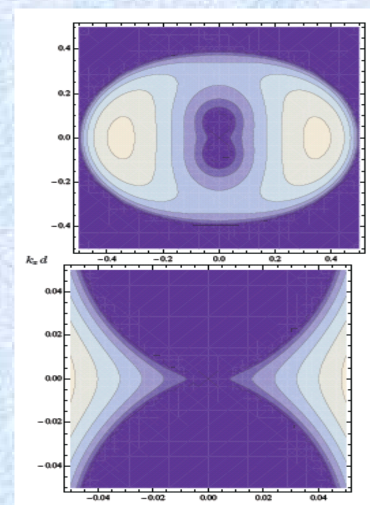
(The dipolar interaction in this limit resembles an extended range XY model).

Stripe order seems to be preferred over checkerboard or hexagonal order for a gas with large spatial extent.

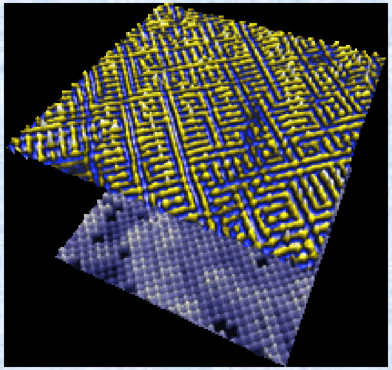


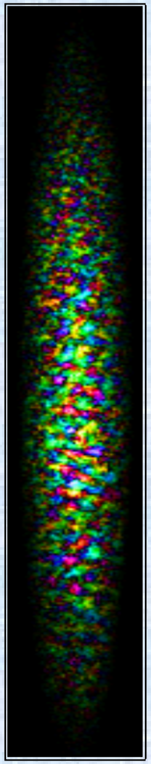


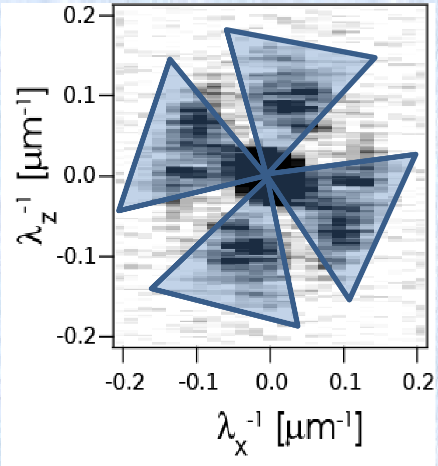
This does not seem to conform with the experiment or with the linear stability analysis.



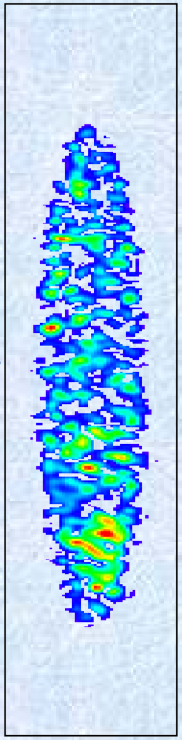
Are we seeing stripe order with two preferred orientations?







X



A Fourier reconstruction of the magnetic order is not consistent with 'mutually repelling' stripe modulations.

Current understanding of the experiment

- Dipolar spinor gases – A rich system capable of a variety of instabilities and quantum phases.
- Observe spontaneous magnetic ordering in the spinor gas due to the interplay between the dipolar and contact interactions.
- This spatial organization seems to coexist with superfluid order indicating a possible supersolid phase.

Are we seeing the true ground state of the dipolar spinor?

✓ Similar checkerboard magnetic order seen for different initial conditions.
✓ Spatial structure appears stable within timescales of the experiment.
✓ Length scales qualitatively agree with a picture of competing phases induced by the dipolar and ferromagnetic interactions.
✓ Fourier reconstruction indicates true two-dimensional spatial modulation, and not orthogonal stripe order.

- Three body lifetime limits evolution time to on the order of 1 second.
- Do not observe true long range spatial order – … fluctuations, finite temperature, finite size?
- Energetics seem to favor stripe order.
- Cannot rule out glassy dynamics.

… Still an open question.

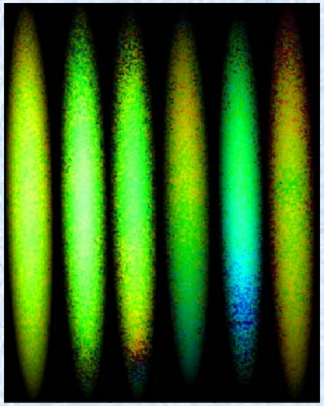
More open questions ...

Ground state of a dipolar quantum fluid?

• *Are we seeing the true ground state of the quasi-2D dipolar spinor gas?*

Transport and flow of this quantum fluid?

• *Can we observe frictionless flow?*
• *Elementary excitations of the spin lattice?*



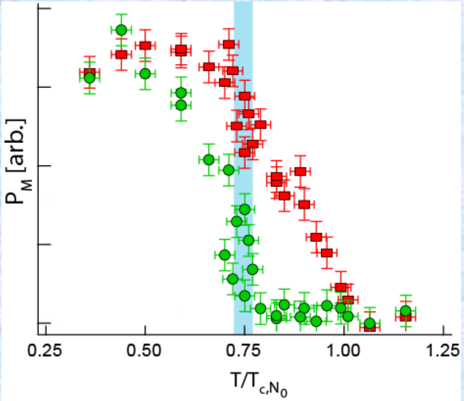
Quantum dynamics

• *Growth of magnetically ordered phase.*

Physics of quantum fluids with competing interactions. What insights can be drawn from this dilute, weakly interacting system?

Long-range crystalline order?

• *Can this 2D solid show true long range order?*



Defect-induced quantum melting?

• *Why do we see the intermediate transition for the (1/4,1/2,1/4) gas?*
• *How does the checkerboard phase undergo a melting transition?*
• *Can we observe a hexatic phase?*

... much to be done.