Erich Mueller
Cornell University
June 8, 2009

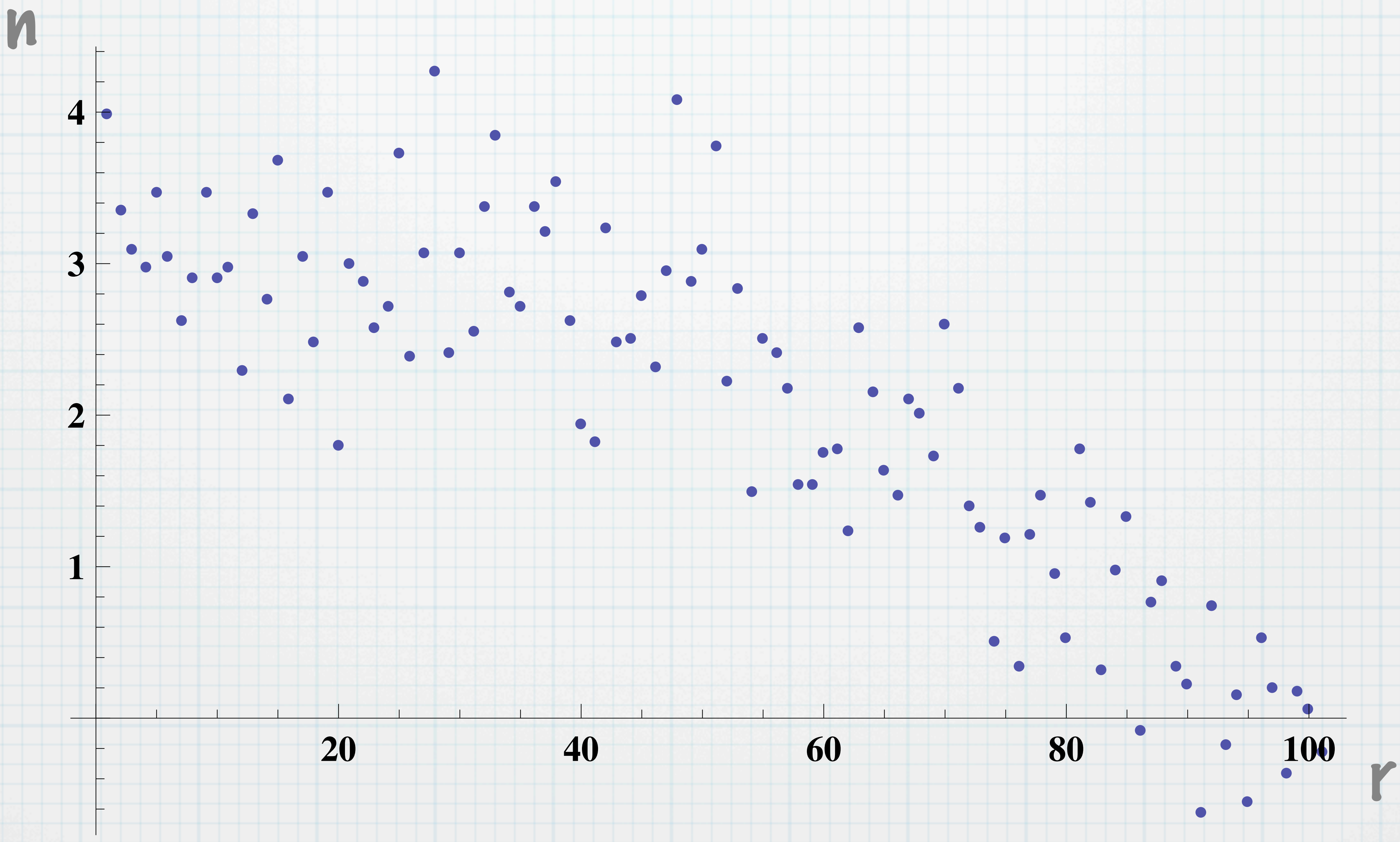
Maximum Entropy

Method for extracting information from noisy data

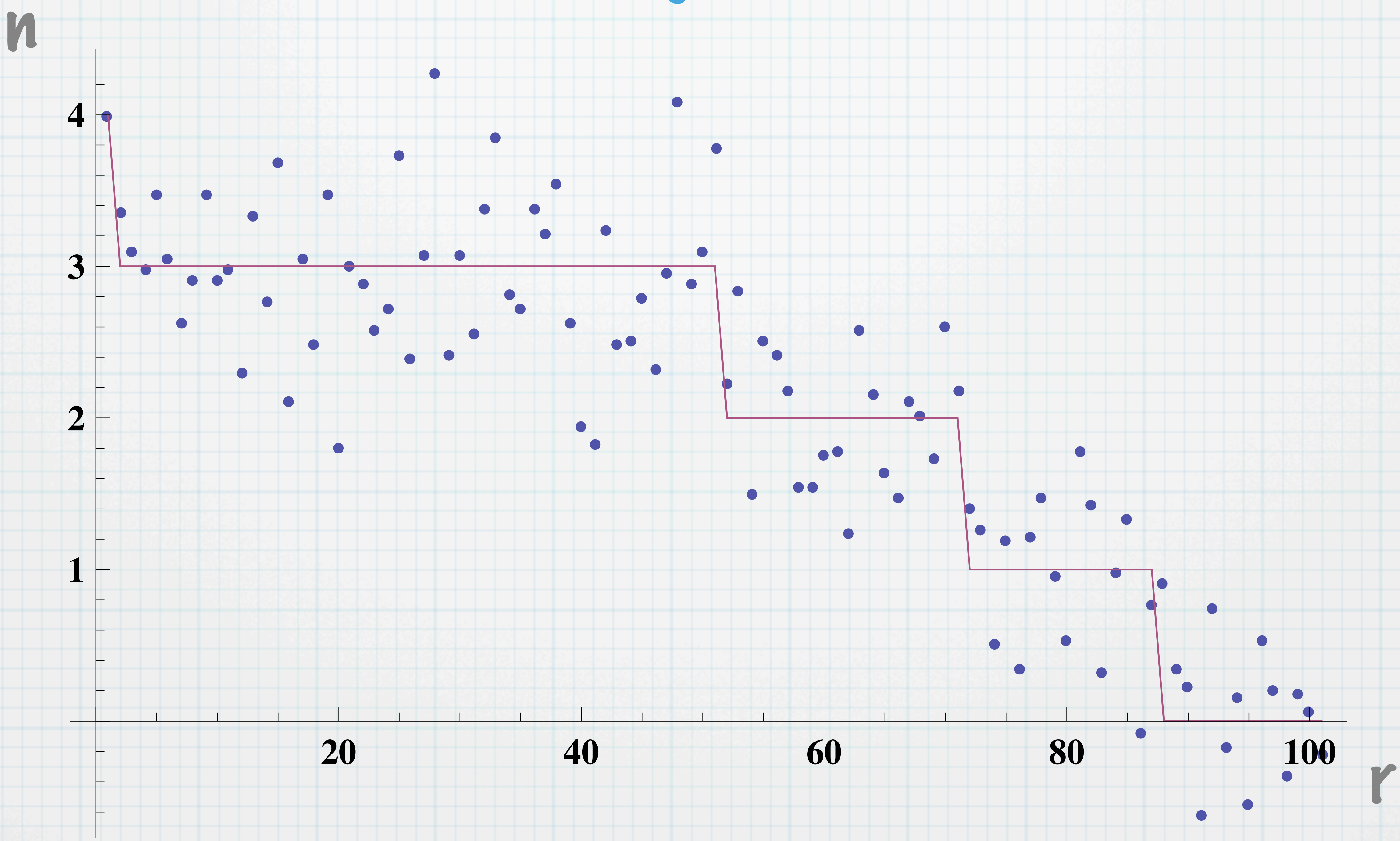
Finding Compressibilities

Reconstructing 3D densities

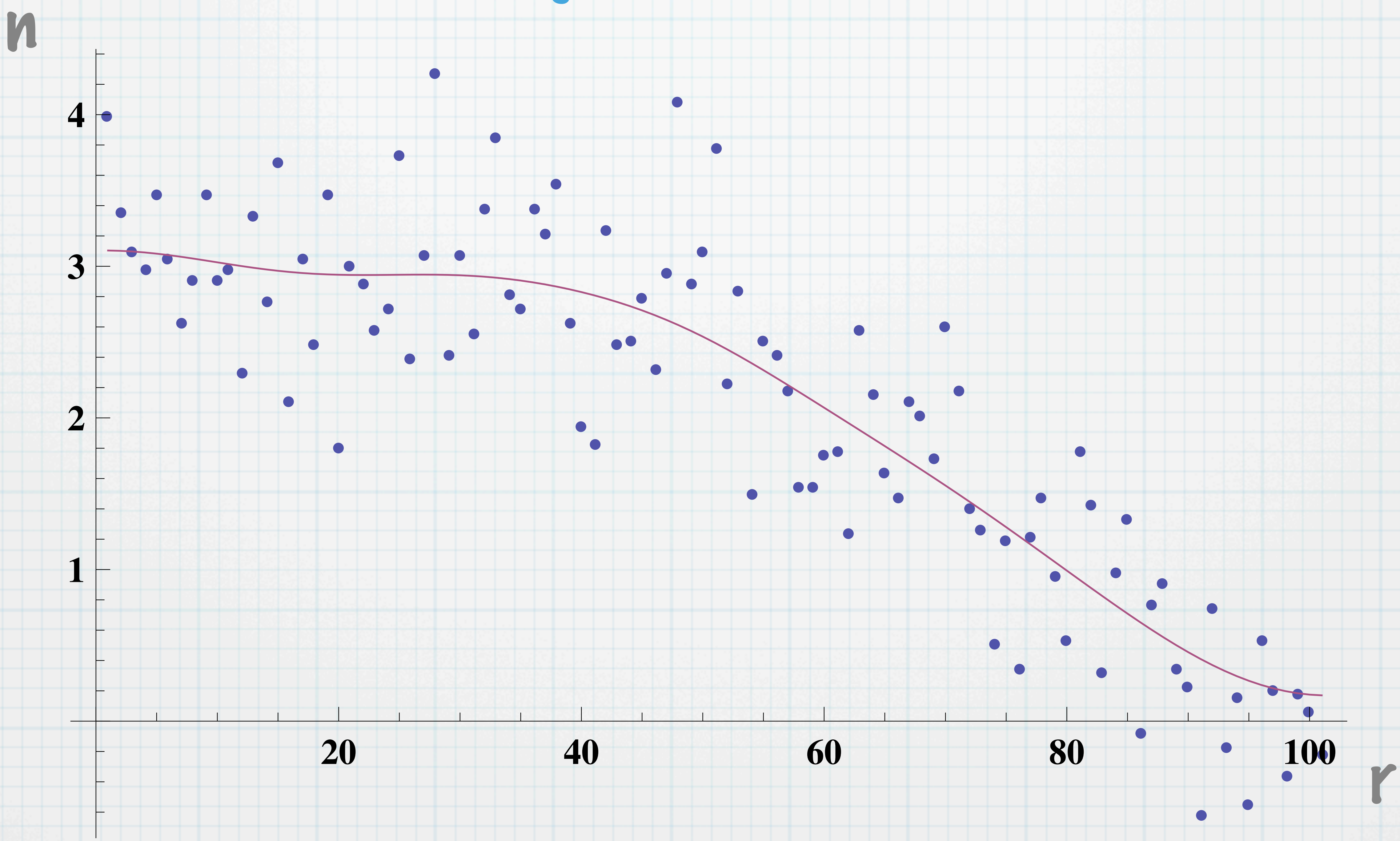
Mott Shells?



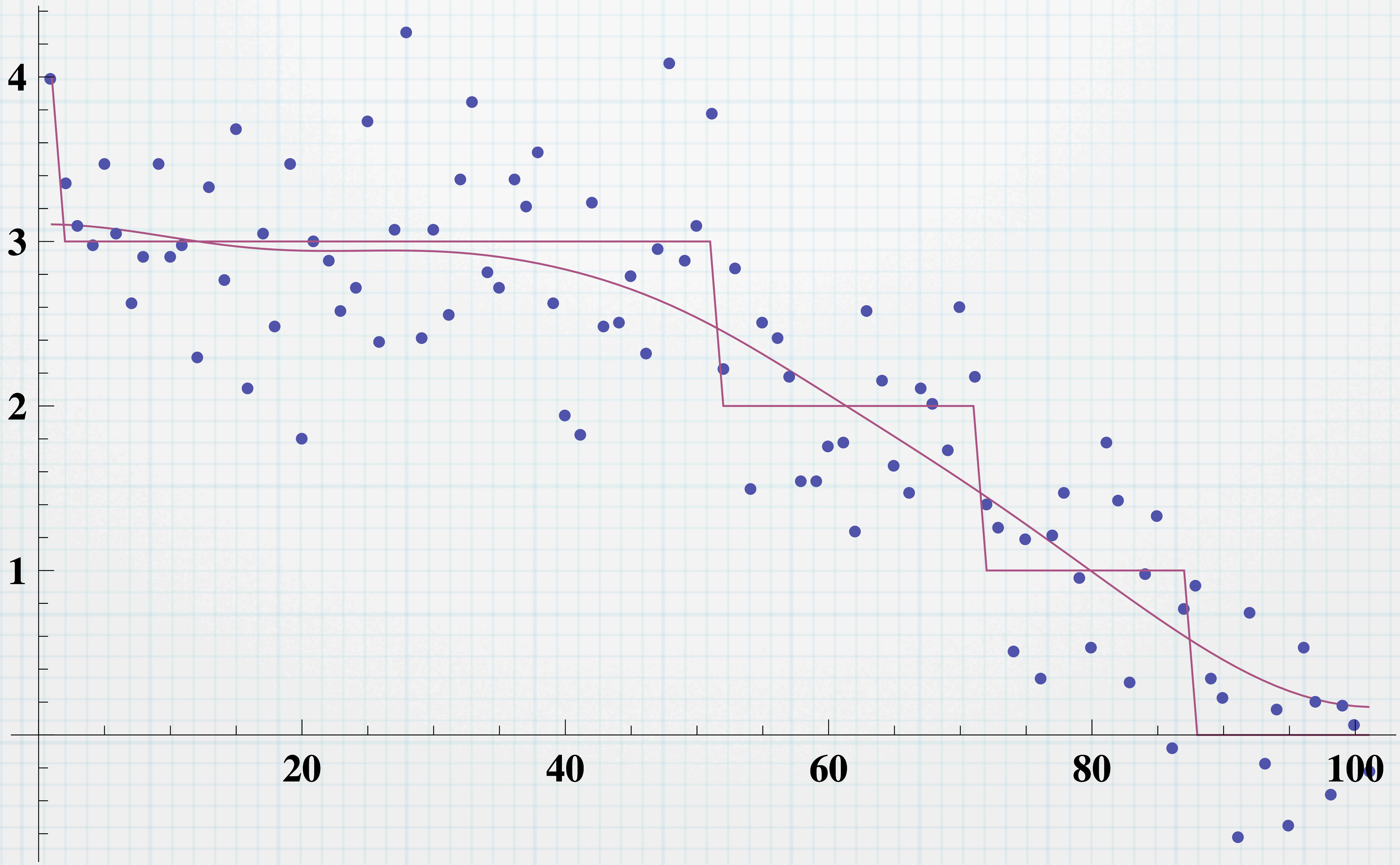
Maybe?

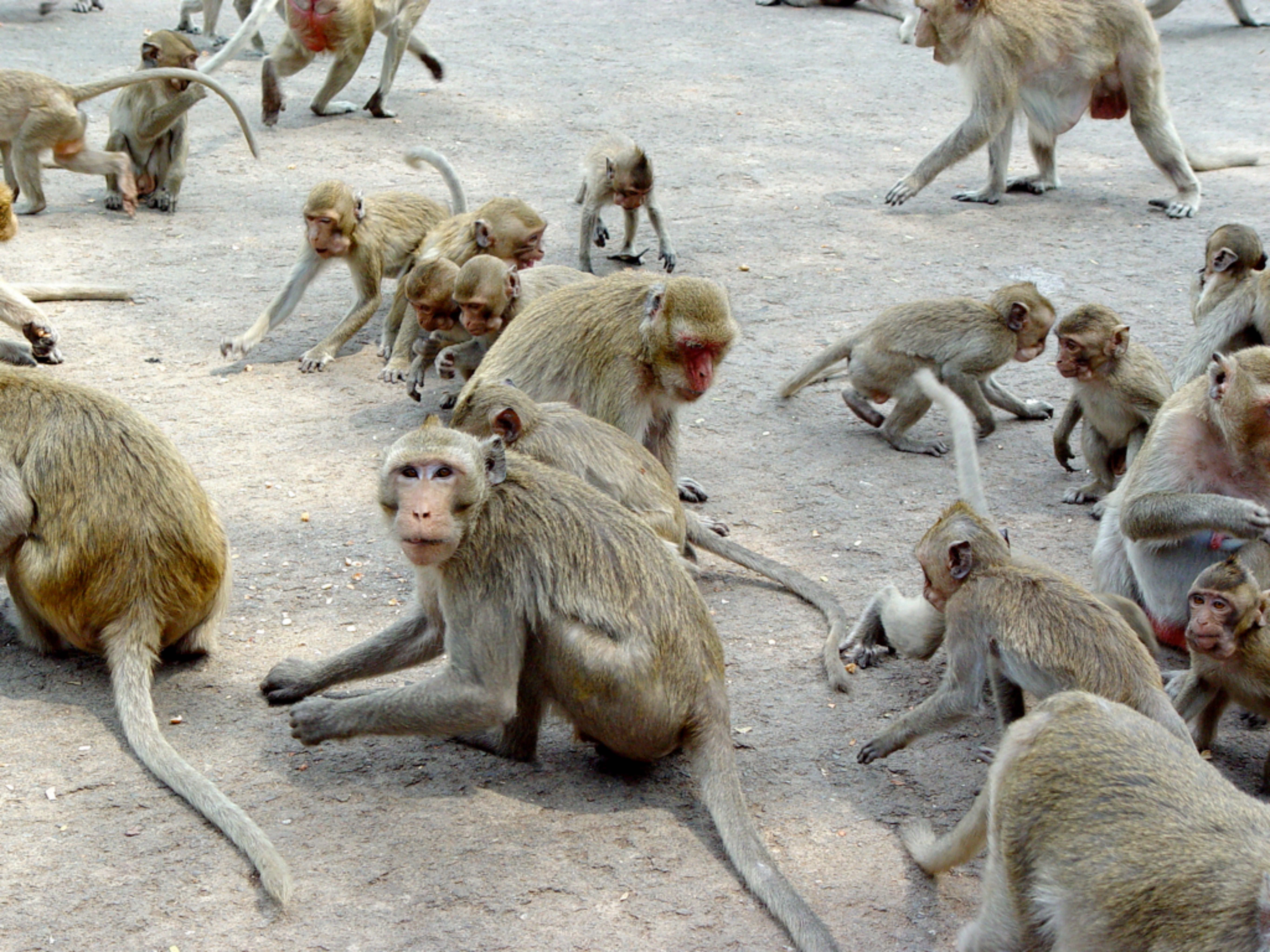


Maybe not?



How to tell?





Monte-Carlo Approach

Generate smooth random data which is consistent with experimental data

Fraction of random data that has Mott plateaus gives probability that there is a Mott plateau

Maximum Entropy

Extension: Can produce most probable smooth reconstruction

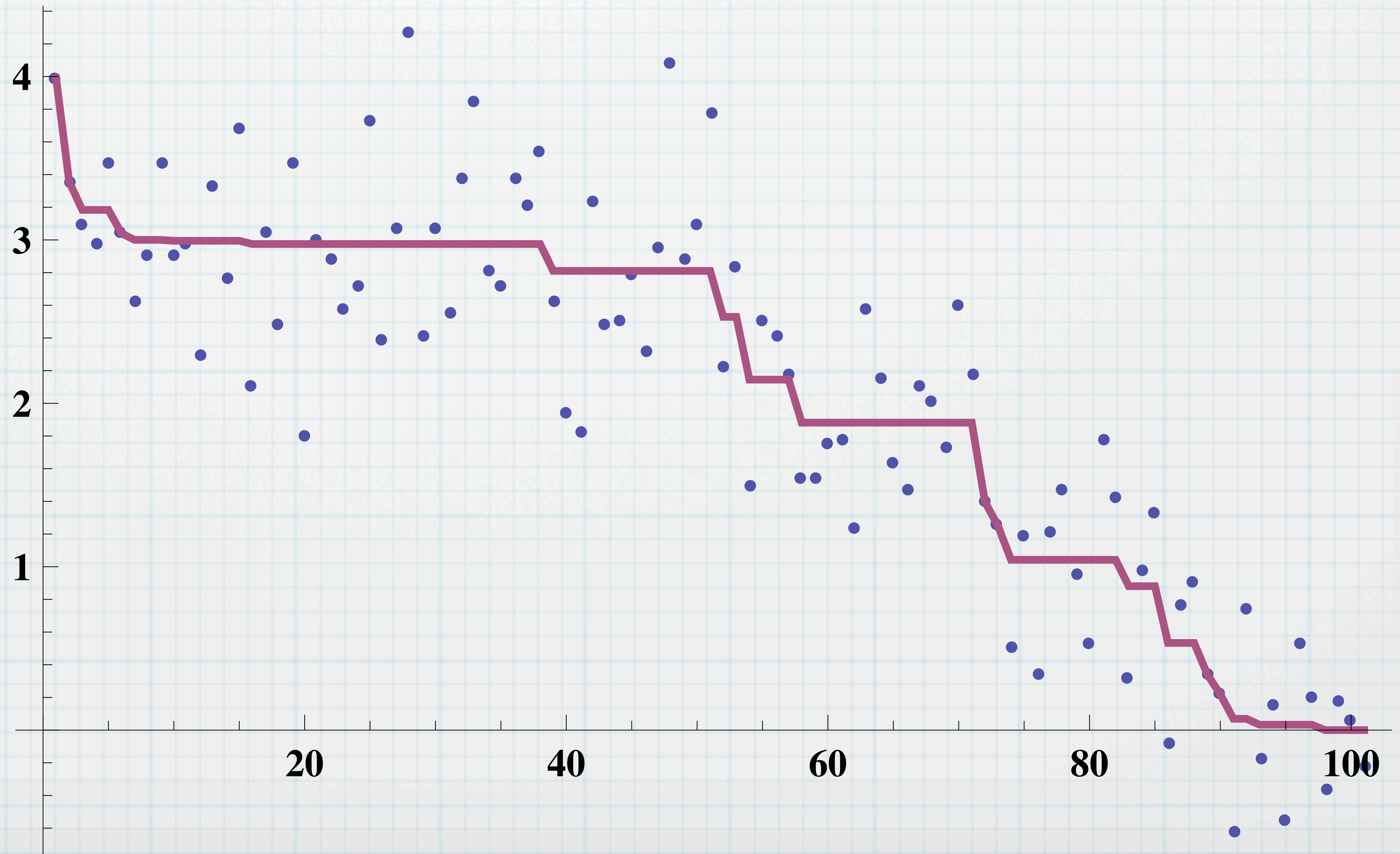
bin consistent random data

bin with most elements = most probable

Tricks

Enforce known symmetries/constraints:

Density is positive
slope is negative



Best Fit
with these
constraints

$$\chi^2 = 0.87$$

(slightly overfit)

Statistical Mechanics

χ^2 plays roll of energy

Microcanonical:

Generate all configurations of fixed χ^2

Canonical:

Minimize Free energy

$$\chi^2 - TS$$

$$\frac{1}{T} = \frac{\partial S}{\partial \chi^2}$$ = Lagrange Multiplier

Choose so fit is appropriate

Entropy

Correct entropy depends on algorithm used to randomly generate data

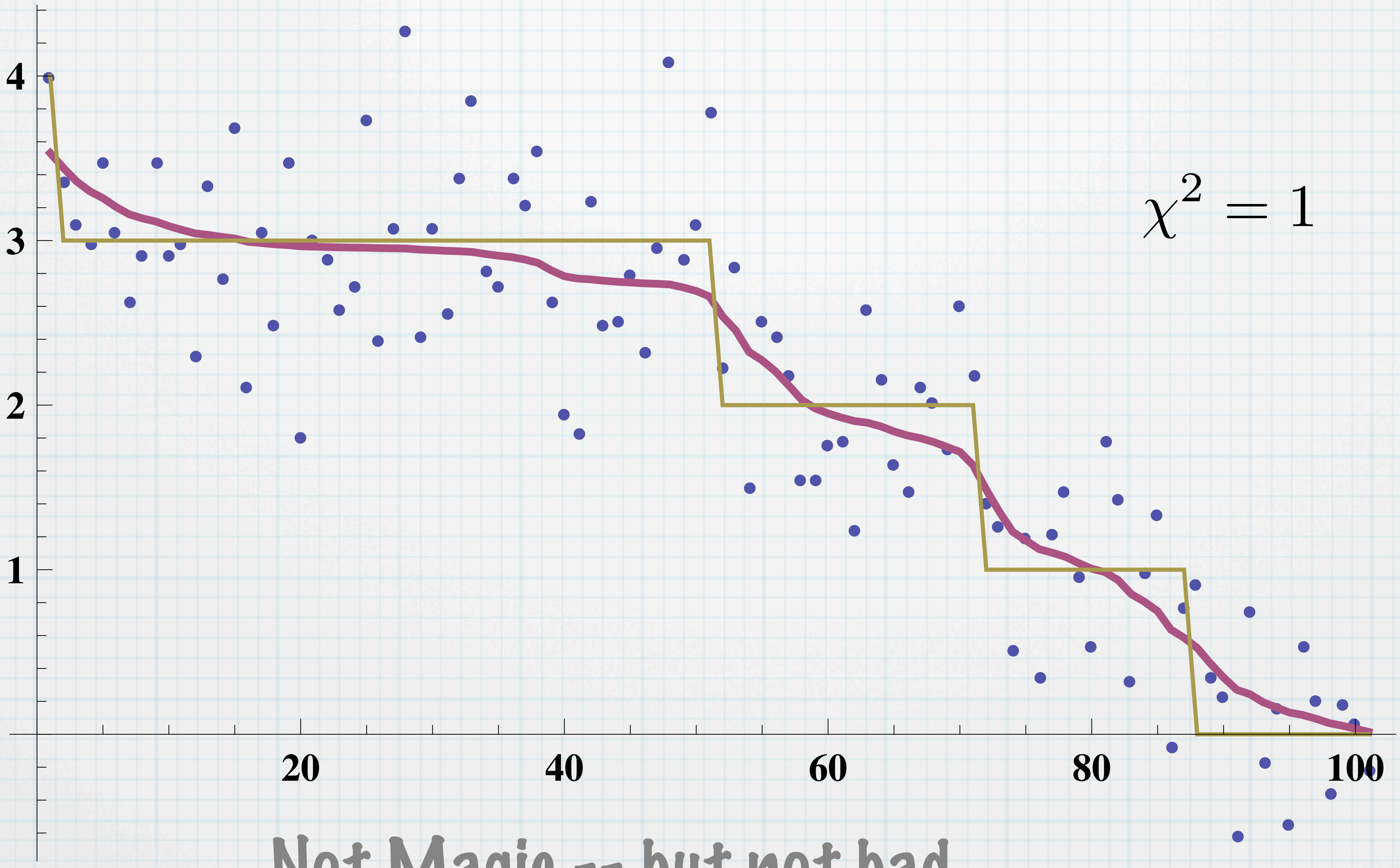
Typical Choice: Shannon Entropy

$$S = -\sum_j \frac{n_j}{N} \log \frac{n_j}{N}$$

$$N = \sum_j n_j$$

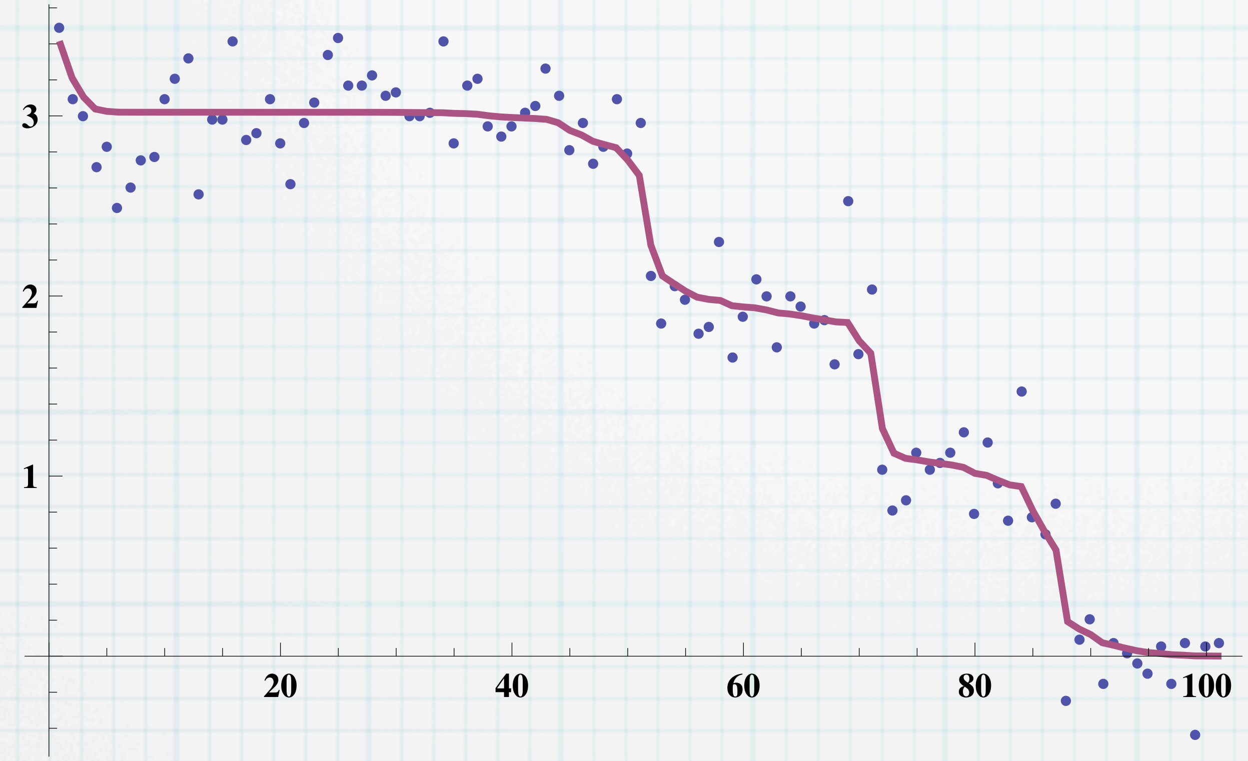
(I take nj= slope at position j)

Results

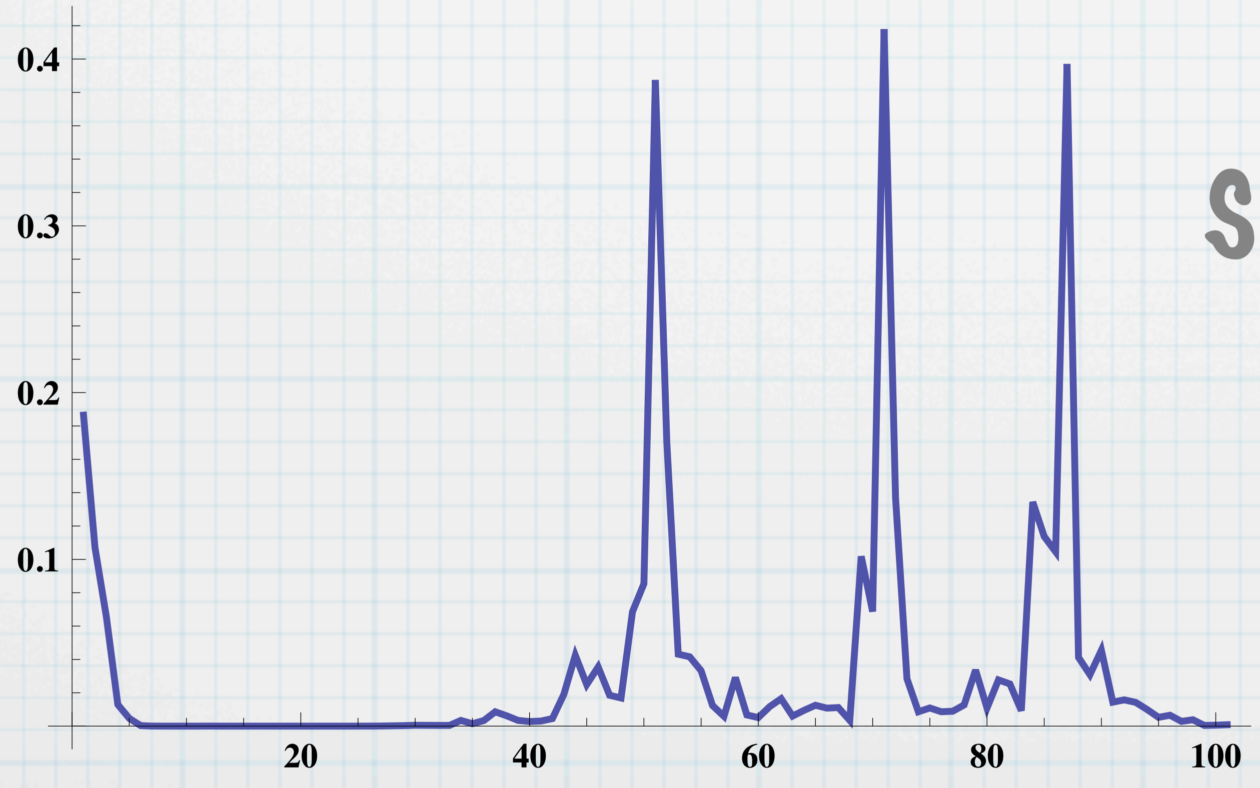


Not Magic -- but not bad

Better Data



Not biased against sharp features



Slope

Abstract Picture

Model Space — s_i — slopes

↓ map — trapezoid rule

Data Space — d_i — density

data: n_i

noise: σ_i

want model that describes data

Model Space → s_i slopes

↓ trapezoid rule

Data Space → d_i density

data: n_i noise: σ_i

Fitting:

$$\chi^2 = \sum_i \frac{(n_i - d_i)^2}{\sigma_i^2}$$

minimize wrt s_i → finite difference derivative

overfit: $\min(\chi^2) = 0$

Model Space → **Data Space**

s_i slopes

↓ trapezoid rule

d_i density

data: n_i noise: σ_i

Better:

$$\chi^2 = \sum_i \frac{(n_i - d_i)^2}{\sigma_i^2} = N_{\mathrm{pixels}}$$

but does not uniquely determine s_i

Bayesian Approach

Find s_i

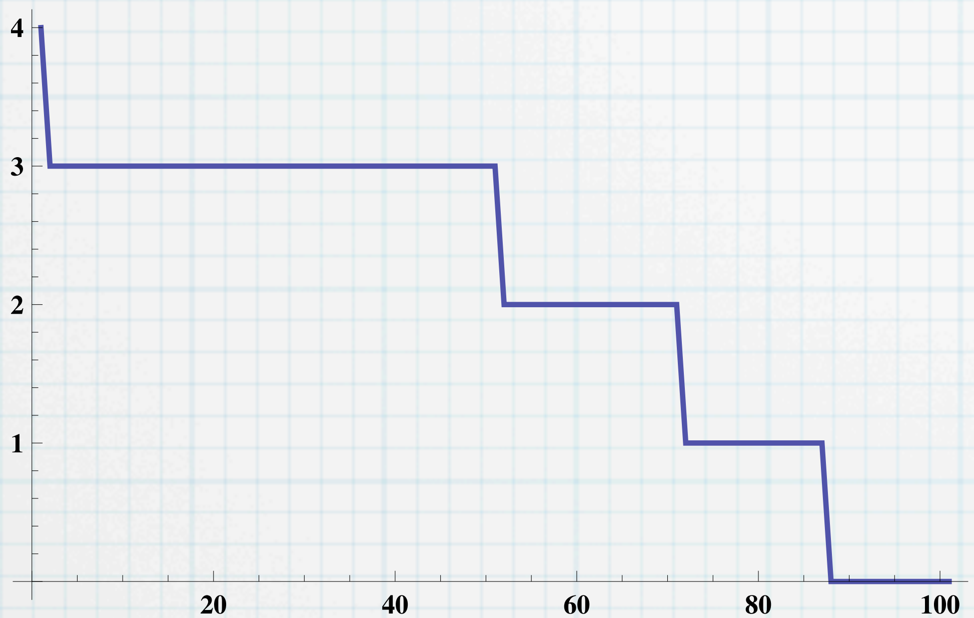
for which

$$\chi^2 = \sum_i \frac{(n_i - d_i)^2}{\sigma_i^2} = N_{\mathrm{pixels}}$$

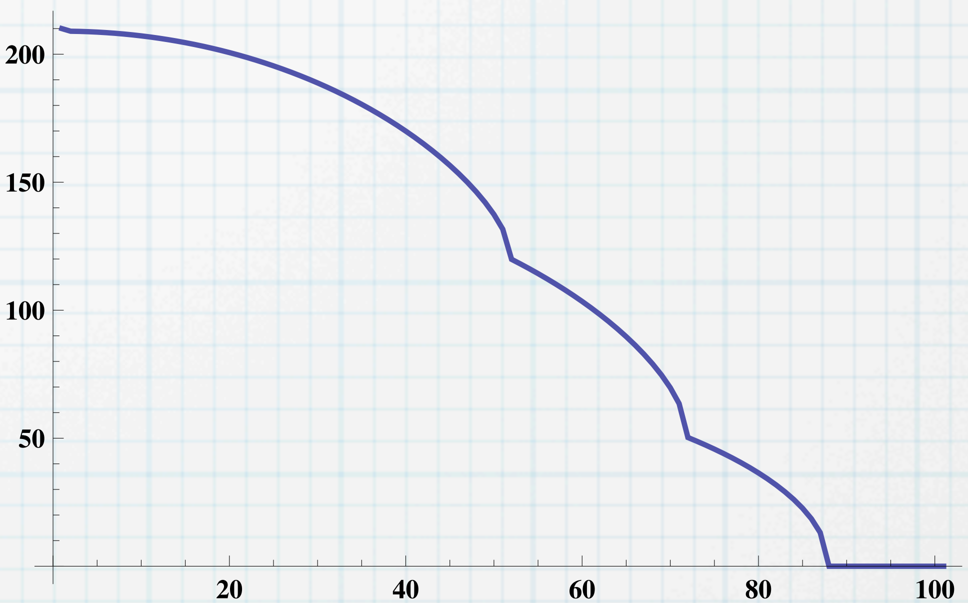
that carries least information

i.e. Maximize Entropy

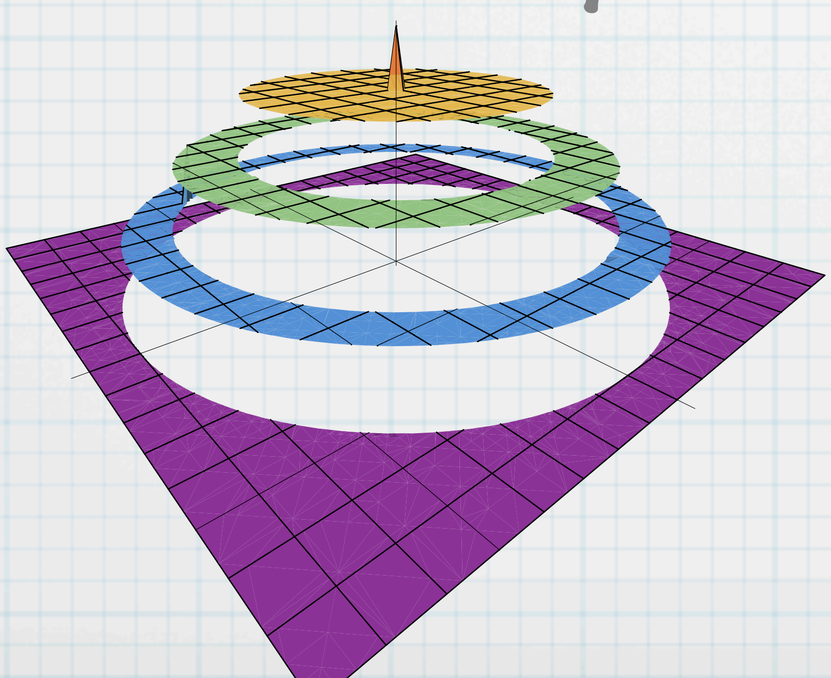
Inverse Abel



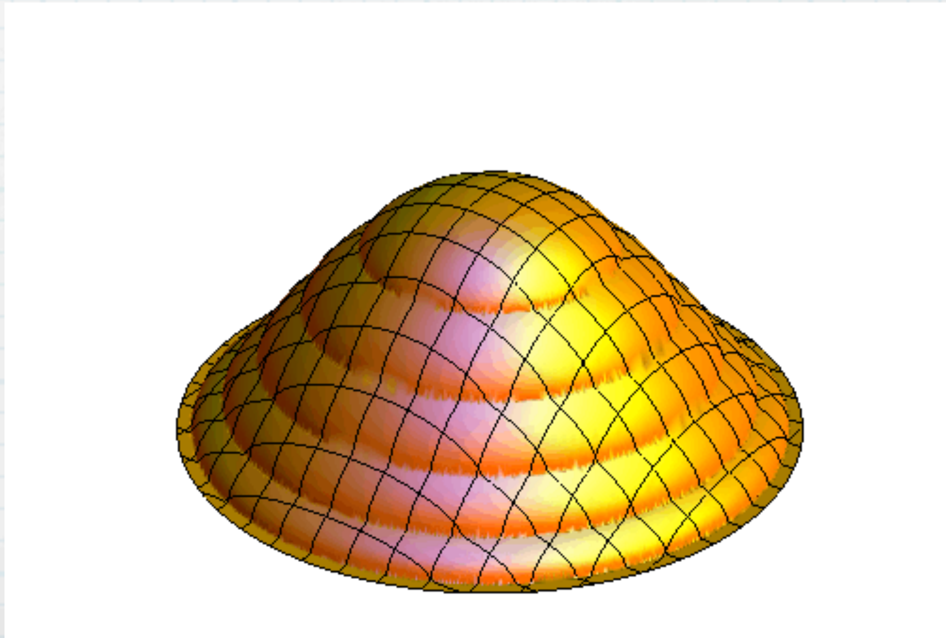
Abel



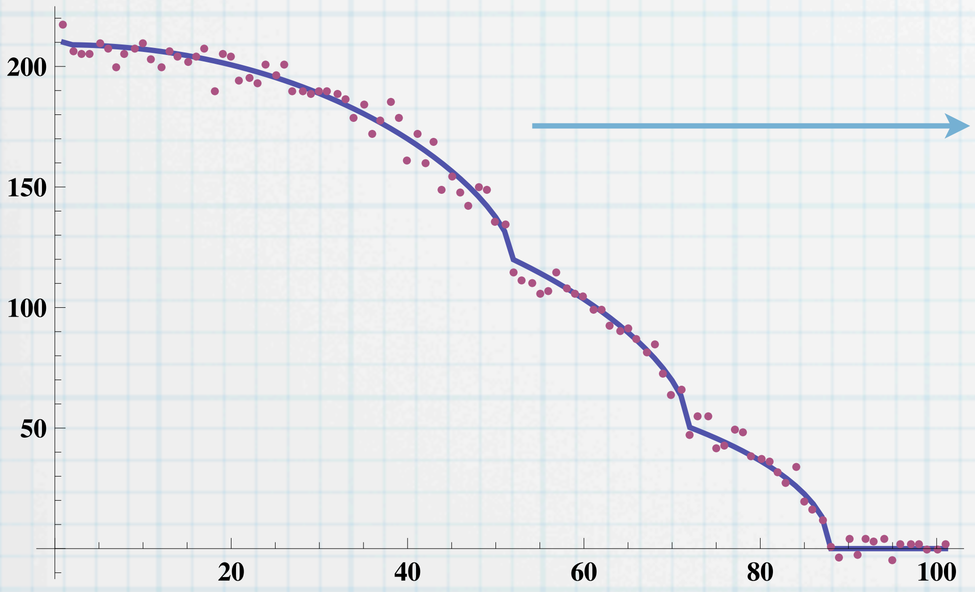
3D Density

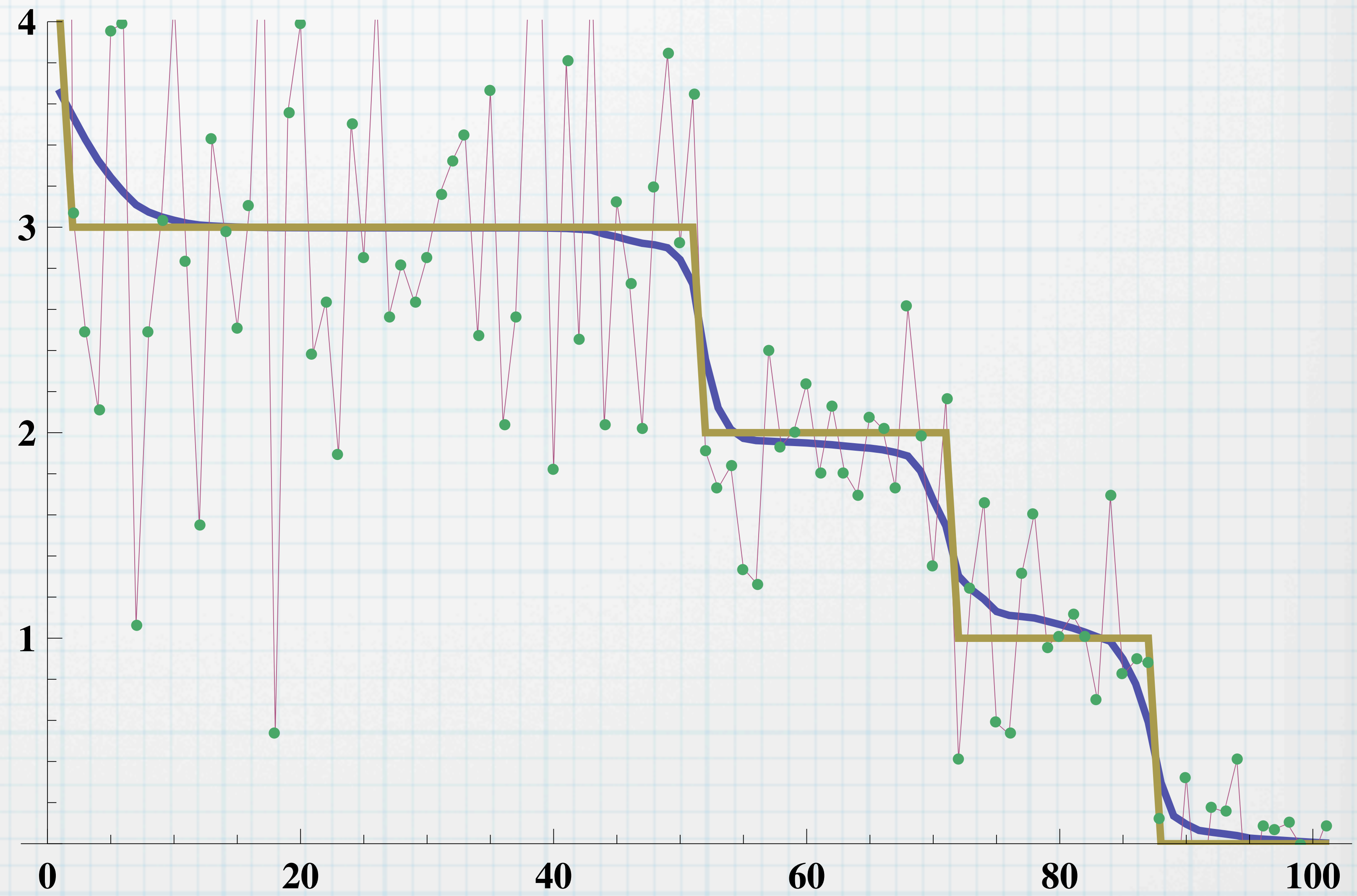


2D Column Density

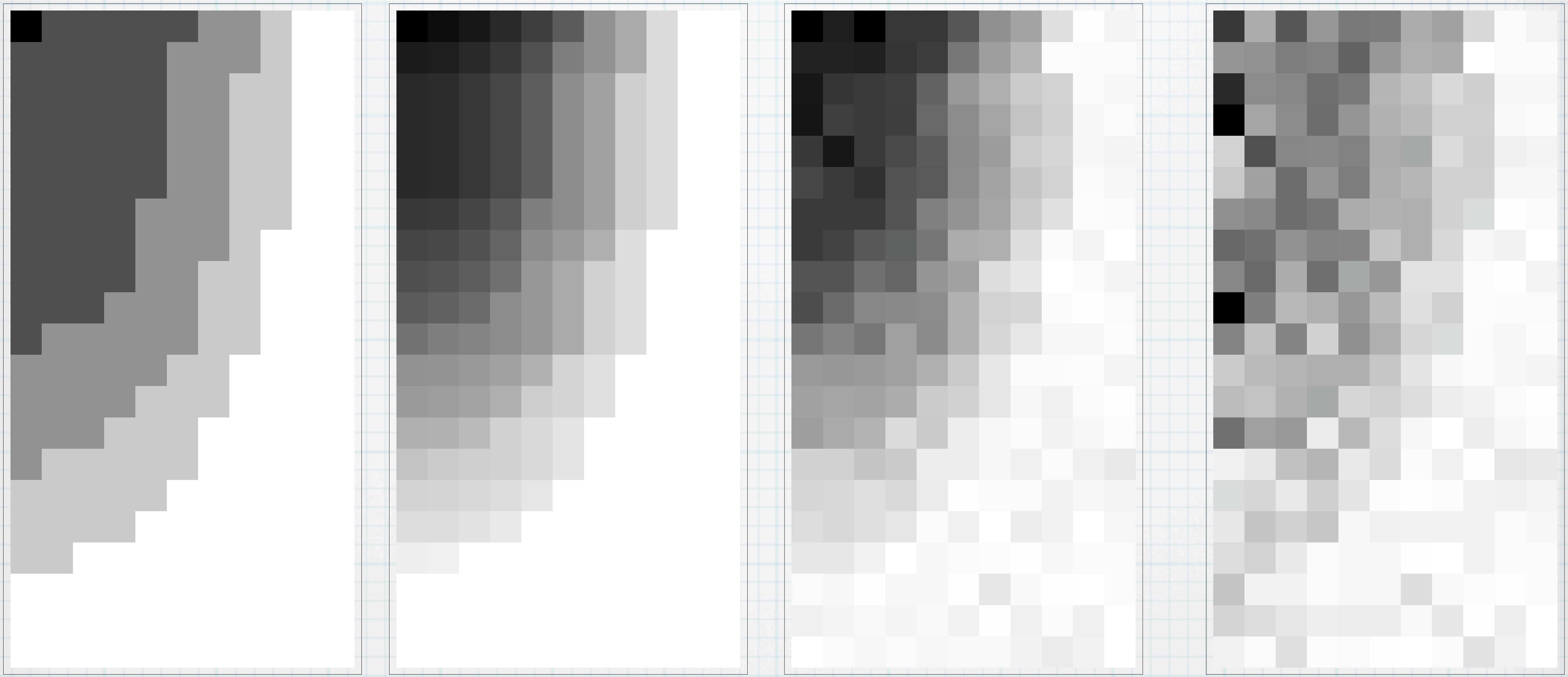


Noisy Data



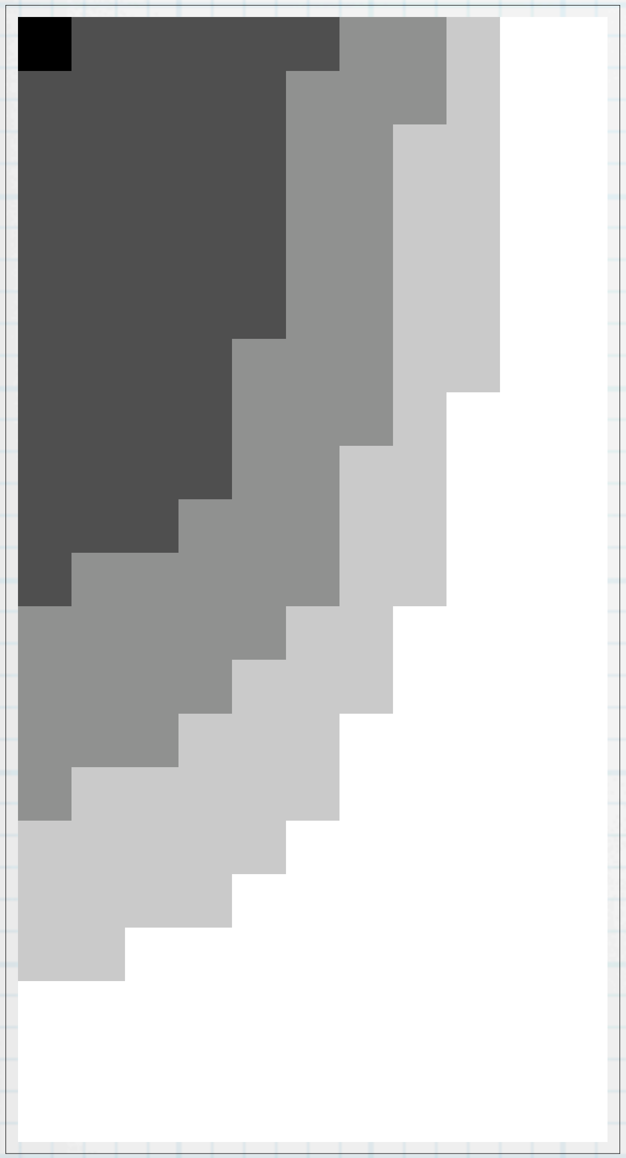


2D



3D density　Column　Noise　Inverse (Naive)

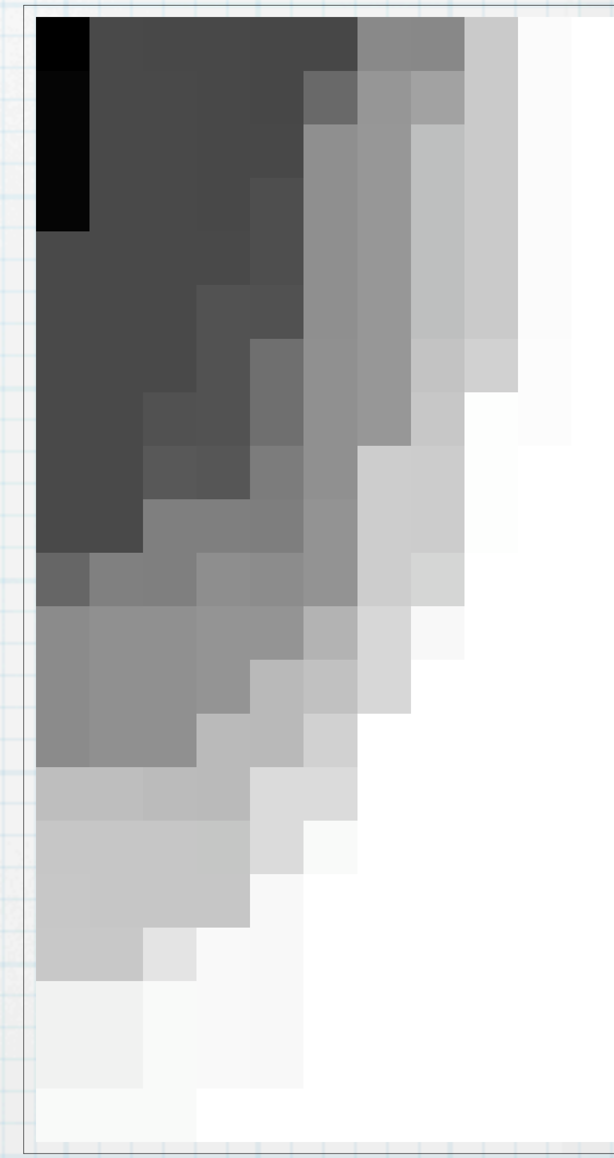
2D



3D density

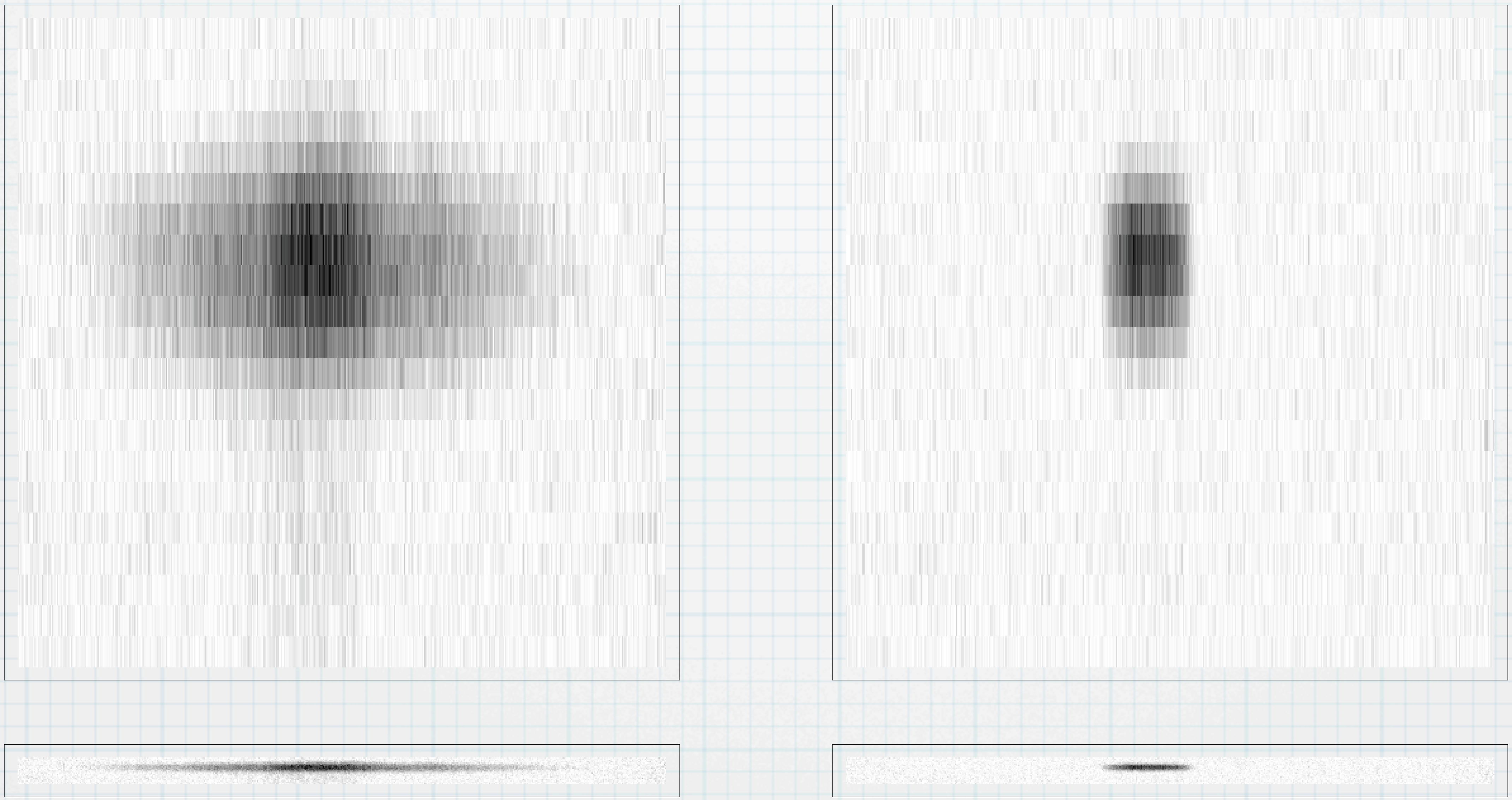
Column

Noise



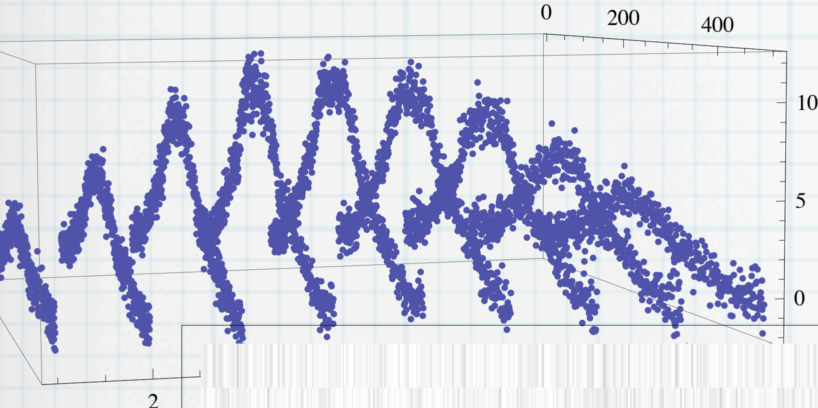
Inverse (Max Ent)

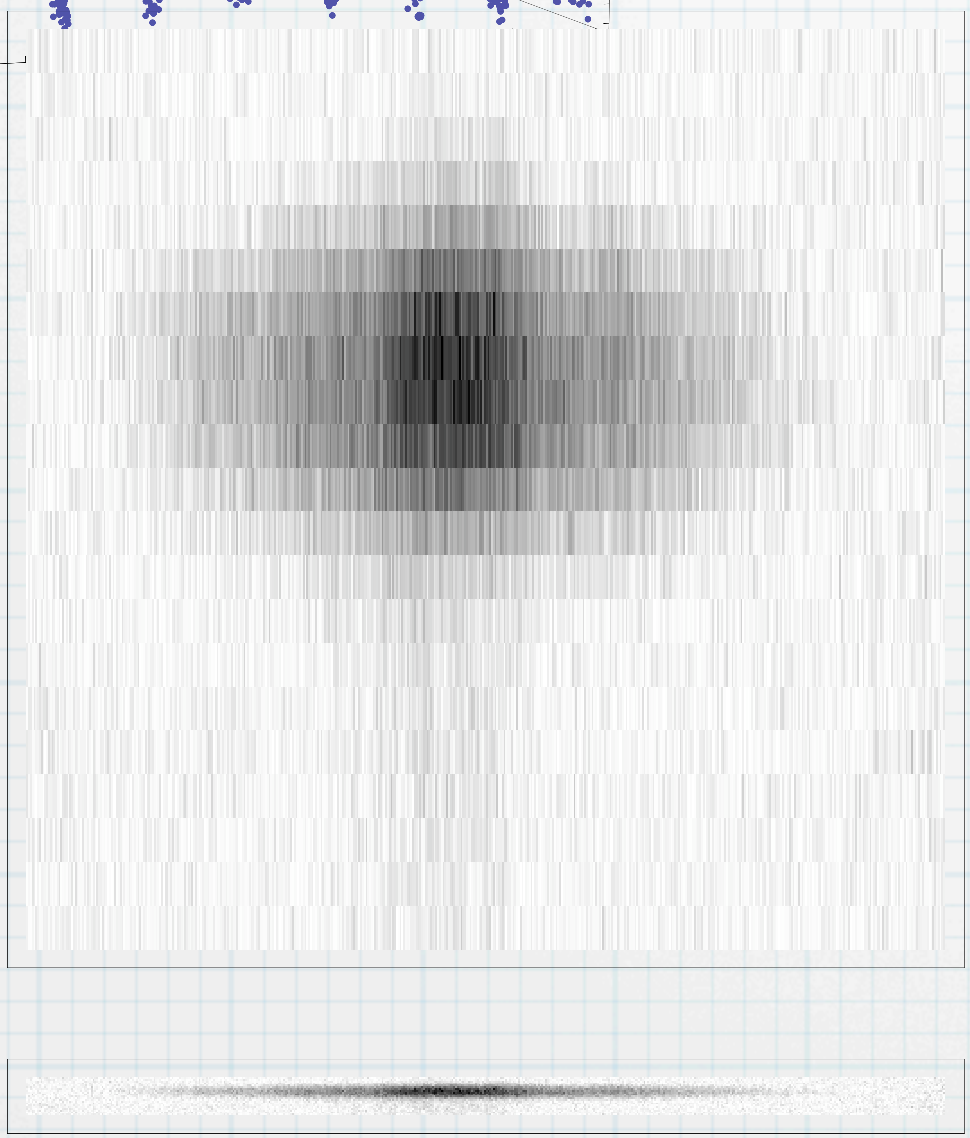
Actual Data

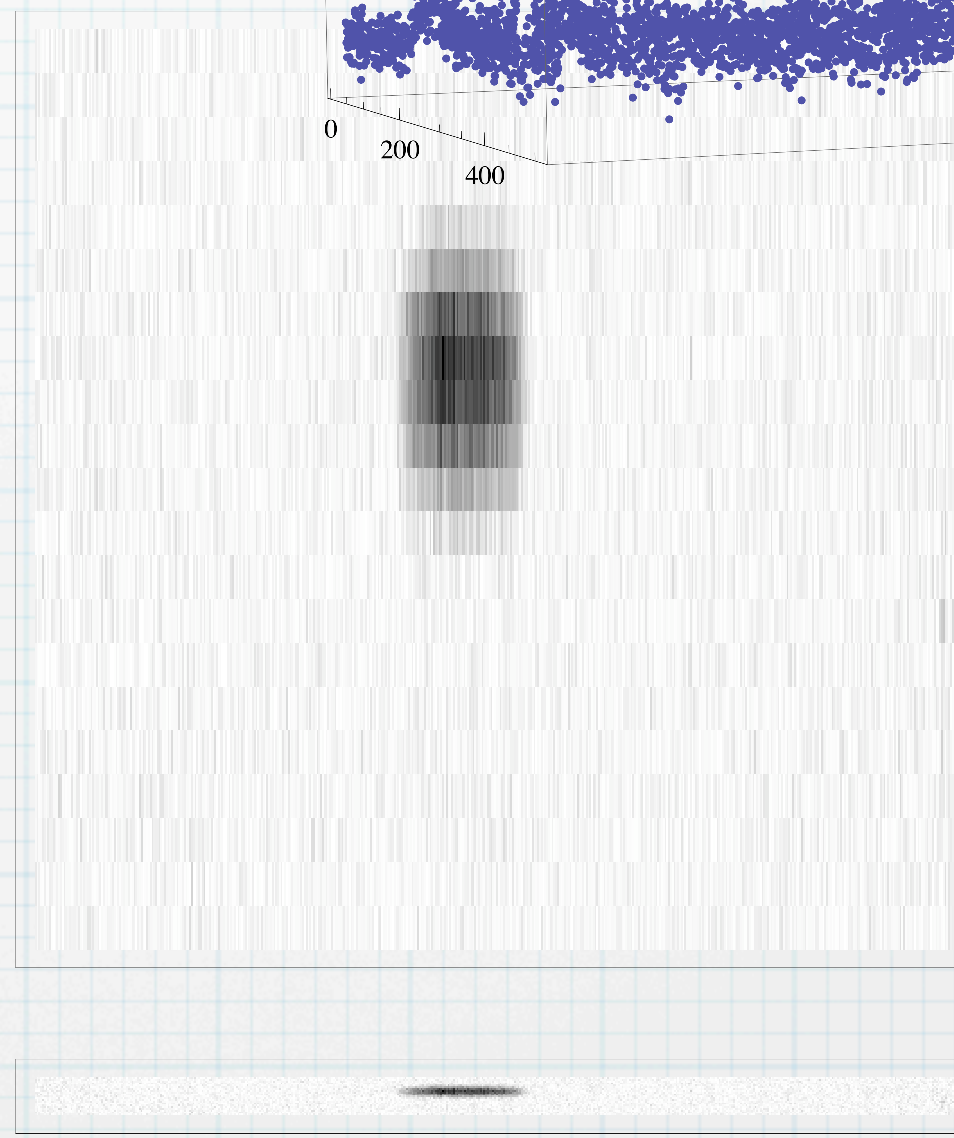


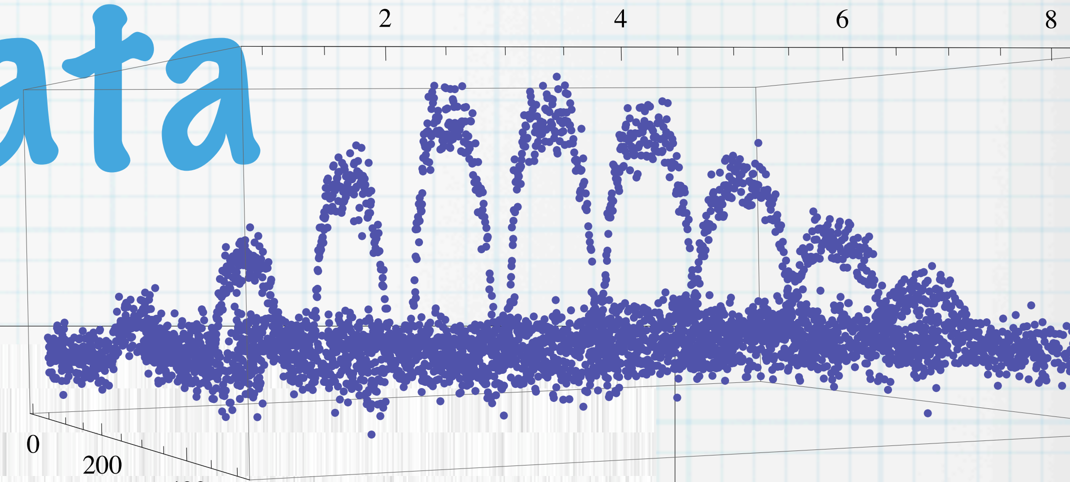
3D -- Unitary Fermi gas -- courtesy of Randy Hulet

Actual Data



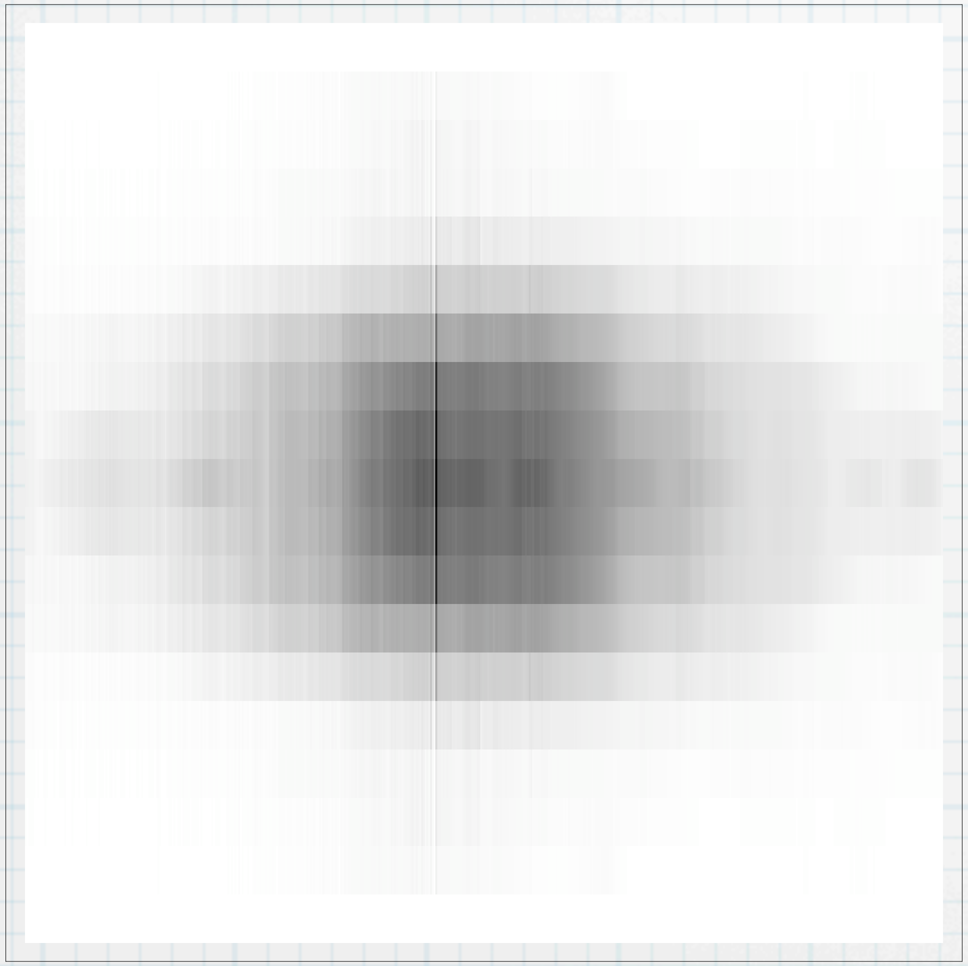




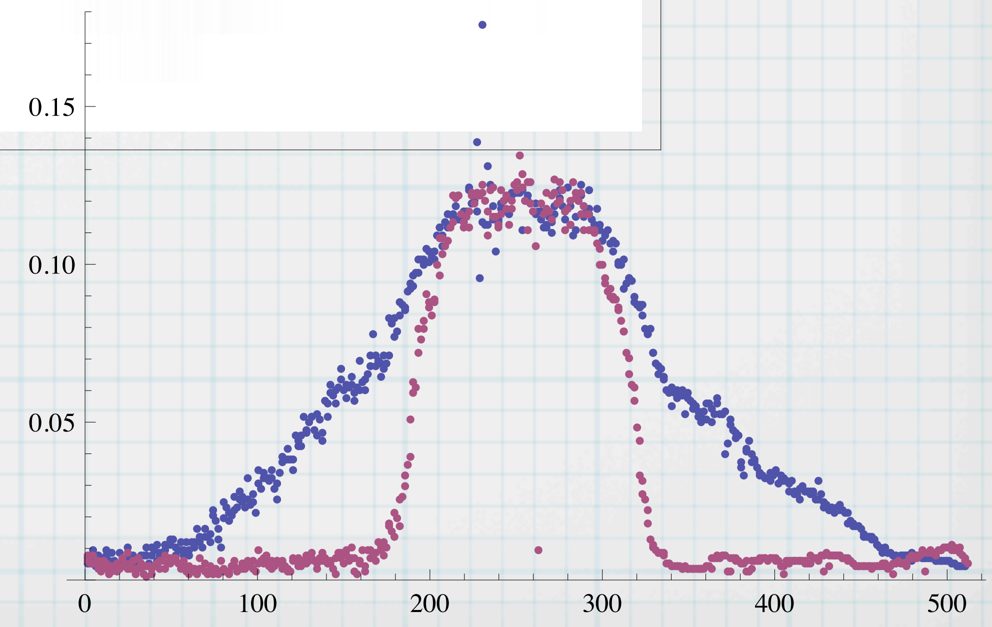


3D -- Unitary Fermi gas -- courtesy of Randy Hulet

3D Reconstruction



off-axis slice:



Thoughts

* Not as good as a real model
* Introduces less bias

* Needs relatively clean data

Dreams: extract equations of state, phase diagrams, entropy (temp), superfluid density...