HW Question
Basic Training in Condensed–Matter Theory
Craig Fennie; Due May, 2009 (talk with Erich)

The linear magnetoelectric effect vs. weak ferromagnetism.

Lets consider the real materials, Cr_2O_3 and Fe_2O_3. Both form in the Corundum (or if you want to sell it, "Sapphire") crystal structure, Space group R$\bar{3}$c. Attached shows a simplified view of the crystal structure and the relevant crystal symmetries. For each case:

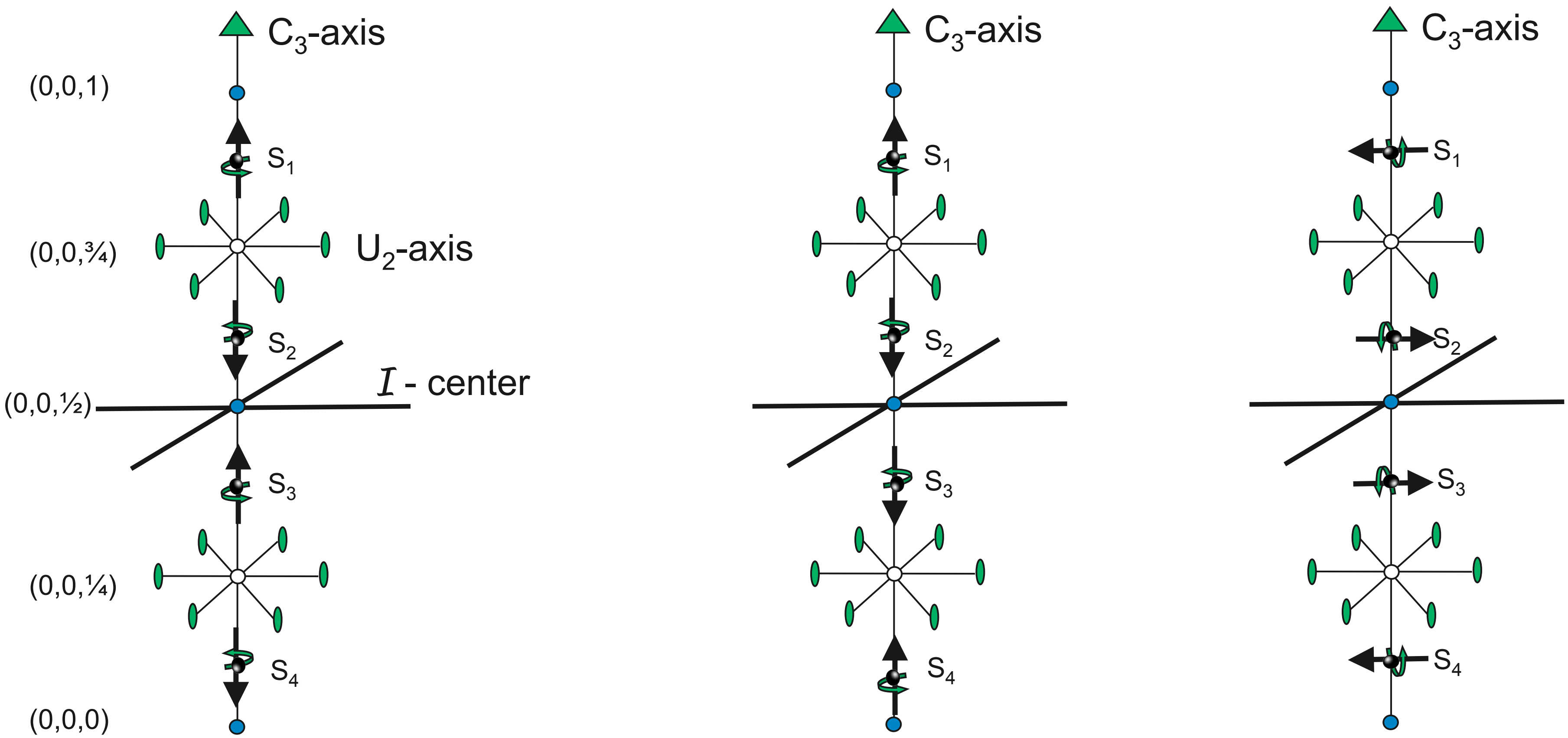
- (a) What are the generators of the magnetic point group (take the generators of $\bar{3}$m and combine with time reversal symmetry, which operations leave the system invariant.)
- (b) Determine whether the system displays a non-zero linear magnetoelectric effect, weak ferromagnetism, or both (teaser: in general, can a system simultaneously display both the linear magnetoelctric effect and weak ferromagnetism? If so when)
- (c) for the system that displays a linear ME effect, what are the non-zero components of the ME tensor.)
- (d) write down the simplest Landau theory for each case (no higher than quadratic) for each system in terms of **P, L, M**.

Note, weak ferromagnetism can be thought of arising from terms like $E_{wFM} = D_{ij}L_iM_j$ (typically arising from a Dzyaloshinskii-Moria type of interaction $E_{wFM} = D\cdot(L\times M)$) whereas the linear magnetoelectrc effect $E_{ME} = \gamma_{ijk}P_iL_jM_k$ where D is an axial vector and P is a polar vector.

A great source for to read up on this HW is Landau and Lifshitz, Electrodynamics of Continuos Media. Also E.A. Turov, "Can the magnetoelectric effect coexist with weak piezomagnetism and ferromagnetism," *Physics - Uspekhi* **37** (3), 303 (1994).

Weak ferromagnetism vs ME effect

Space group: R\bar{3}c #167
Point group: \bar{3}m (generators: C_3, $\mathcal{I}$, U_2)



$L_a \equiv (S_1 - S_2 + S_3 - S_4)$ z-hat

$L_b \equiv (S_1 - S_2 - S_3 + S_4)$ z-hat

$L_b \equiv (S_1 - S_2 - S_3 + S_4)$ x-hat

Spins are at Wyckoff position 3c: (0,0,z) (0,0,-z+½) (0,0,-z) (0,0,z+½)