

Teaser Question 3

Basic Training in Condensed-Matter Physics

Erich Mueller; Due Wednesday, Feb 25, 2009

Response Function Exercise This is not a traditional teaser, but it is a good exercise.

Consider a harmonic oscillator $H = p^2/2 + x^2/2$ with $[x, p] = i$. Assume at $t = 0$ the system is in thermal equilibrium with temperature T .

(a) What are $\langle x(0)^2 \rangle$, $\langle p(0)^2 \rangle$, $\langle x(0)p(0) \rangle$ and $\langle p(0)x(0) \rangle$?

(b) Write the operator $x(t)$ as a function of $x(0)$ and $p(0)$.

(c) Calculate $\chi^R(t) = \frac{\theta(t)}{i} \langle x(t)x(0) - x(0)x(t) \rangle$. Does this depend on temperature?

(d) Calculate $\chi^>(t) = \langle x(t)x(0) \rangle$. Does this depend on temperature?

(e) Calculate $\chi(t) = \frac{\theta(t)}{i} \langle x(t)x(0) \rangle + \frac{\theta(-t)}{i} \langle x(0)x(t) \rangle$.

(f) Sketch $\chi^>(\omega)$ and $\text{Im}\chi^R(\omega)$.