P653 HW5

Due Sept 29, 2005

Problem 1. Upper Critical Dimension

By comparing the fluctuations in one coherence volume to the mean field, calculate the upper critical dimension for a system with free energy

$$f = am^2/2 + bm^6,$$

where m is the order parameter. As usual, we assume that a changes sign at the critical point and b is positive. Such a free energy corresponds to a tricritical point.

Problem 2. One dimensional s-state Potts model

Calculate the transfer matrix and free energy of the one-dimensional s-state Potts model with periodic boundary conditions. This is a generalization of the Ising model where the spin on each site σ_j can take on the values $1, 2, \ldots, s$, and has a Hamiltonian

$$H = -K \sum_{j=1}^{N} [s\delta_{\sigma_j \sigma_{j+1}} - 1].$$

Problem 3. Critical Exponents

Here we explore if critical exponents can be different above and below the transition temperature.

We will concentrate the critical exponent for the coherence length, taking $\xi \sim (T - T_c)^{\nu}$ $(T > T_C)$ and $\xi \sim (T_c - T)^{\nu'}$ $(T < T_c)$. As shown in class, the scaling hypothesis tells us that the singular part of the free energy density is

$$f = |t|^{d\bar{\nu}} F_f^{\pm}\left(\frac{h}{|t|^{\bar{\nu} y_h}}\right),$$

with $\bar{\nu} = \nu$ or ν' depending on if $T > T_c$ or $T < T_c$. For $h \neq 0$, f should be a smooth function of $t = (T - T_c)/T_c$, because the only singularity is the one coming from the critical point at $t = h = 0$. Show that f can be written in the form

$$f = h^{d/y_h} \phi_{\pm}\left(\frac{h}{|t|^{\bar{\nu} y_h}}\right),$$

and explain how the smoothness assumption mentioned above constrains the analytic form of the functions $\phi_{\pm}$. Hence show that $\nu = \nu'$.